Sections 5.3: The Fundamental Theorem of Calculus

Definition: The **indefinite integral** of f is used to indicate the process of finding the antiderivative of f. i.e. $\int f(x)\ dx = F(x) + C$

Example: Compute.

A) $\int 2x^5 + 7x + 4\ dx =$

B) $\int 3x^2\ da =$

C) $\int \frac{x^2 + 2x^5 + 7x^3 + 4}{4x^3}\ dx$

The Fundamental Theorem of Calculus, Part 2 If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\ dx = F(b) - F(a), \qquad \text{where } F \text{ is any antiderivative of } f.$$

Example: Compute the following.

A) $\int_1^5 3x^2 + 4x + 2\ dx =$

B) $\displaystyle\int_{0}^{4} 3x + 8e^{4x}\ dx =$

C) $\displaystyle\int_{-2}^{5} \frac{1}{x^2}\ dx =$

D) $\displaystyle\int_{0}^{3} |x^2 - 4|\ dx =$

E) $\int_{4}^{9} \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 \, dx =$

Example: Sketch the region enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 4$ and calculate its area.

The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\ dt, \qquad \text{with } a \leq x \leq b$$

is continuous on $[a, b]$ and is differentiable on (a, b), and $g'(x) = f(x)$

Example: Find $g'(x)$.

A) $g(x) = \int_a^x t^2 + 1\ dt$

B) $g(x) = \int_4^{x^2} \tan^3(t)\ dt$

C) $g(x) = \int_{x^3}^{2} \ln(u)\ du$

D) $g(x) = \int_{x^2}^{x^3+1} u^5 + 2\ du$

Example: Define $g(a)$ by $g(a) = \int_0^a f(x)\, dx$ where $f(x)$ is the graph given below.

1) Compute $g(10)$ and $g(20)$.

2) Find the intervals where $g(a)$ is increasing.

3) If possible, give the values of the absolute maximum and absolute minimum.