Sections 5.2: The Definite Integral

Definition of a Definite Integral: If f is a function on the interval $[a, b]$, we partition the interval $[a, b]$ into n subintervals of equal width $\Delta x = \frac{b-a}{n}$. Let x_i^* is any value in the ith subinterval. Then the **definite integral of f from a to b** is

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

provided the limit does exist. If the limit does exist, we say f is **integrable** on the interval $[a, b]$.

Note 1: If $f(x) \geq 0$ on the interval $[a, b]$, then the definite integral is the area bounded by the function f and the x-axis from $x = a$ to $x = b$.

Note 2: If $f(x)$ is not always greater than or equal to zero on the interval $[a, b]$, then the definite integral can be interpreted as the net area on the interval.

Theorem: If f is continuous on $[a, b]$, or if f has only a finite number of jump discontinuities, then f is integrable on $[a, b]$; that is, the definite integral $\int_a^b f(x)dx$ exists.

Example: Estimate $\int_0^6 x^2 - 4\ dx$ using a Riemann sum with 3 rectangles with equal bases and the midpoint rule.

Example: Suppose that $R(t)$ is the rate, in gallons per hour, that water is pumped into a pool at a water park. Explain the meaning of these integrals.

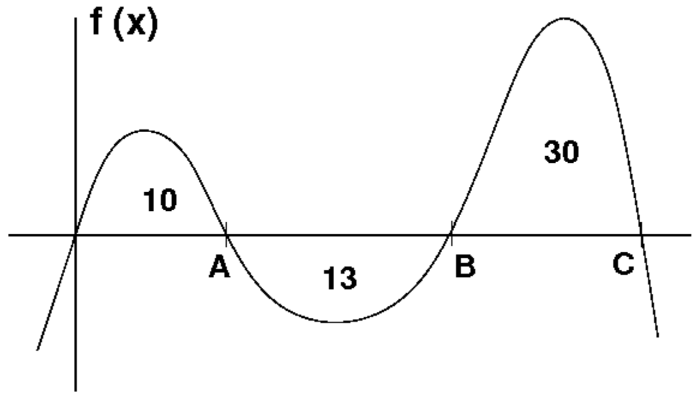
A) $\int_0^5 R(t)\ dt$

B) $\int_3^4 R(t)\ dt$

Example: Use the graph of f along with the indicated areas to compute these definite integrals.

A) $\int_0^A f(x)\ dx =$

B) $\int_A^B f(x)\ dx =$



C) $\int_0^B f(x)\ dx =$

D) $\int_A^A f(x)\ dx =$

E) $\int_A^C 2f(x)\ dx =$

F) $\int_A^0 f(x)\ dx =$

G) $\int_B^A f(x)dx =$

Example: Compute these definite integrals.

A) $\int_{-4}^{4}\ 1 + \sqrt{16 - x^2}\ dx$

B) $\int_0^3\ 2x + 5\ dx$

Properties of Definite Integrals

$$\int_a^b c\ dx = c(b-a)$$

$$\int_a^b cf(x)\ dx = c\int_a^b f(x)\ dx$$

$$\int_a^b f(x) \pm g(x)\ dx = \int_a^b f(x)\ dx \pm \int_a^b g(x)\ dx$$

$$\int_a^b f(x)\ dx = -\int_b^a f(x)\ dx$$

$$\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$$

Example: If $\int_0^3 f(x)dx = 4$, then evaluate $\int_0^3 (5-2f(x))dx$

Example: If $\int_1^5 f(x)\ dx = 6$, $\int_1^{10} g(x)\ dx = 10$, and $\int_1^{10} 3f(x) - 4g(x)\ dx = 35$, then compute $\int_5^{10} f(x)\ dx$.

Comparison Properties of the Integral

1) If $f(x) \geq 0$ for $a \leq x \leq b$, then $\displaystyle\int_a^b f(x)\ dx \geq 0$

2) If $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\displaystyle\int_a^b f(x)\ dx \geq \int_a^b g(x)\ dx$

3) If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then $\displaystyle m(b-a) \leq \int_a^b f(x)\ dx \leq M(b-a)$

Example: Estimate these definite integrals.

A) $\displaystyle\int_2^4 \ln(x)\ dx$

B) $\displaystyle\int_0^{\pi} 2\sin^3(x) + 1\ dx$