Section 4.4: Indeterminate Forms and L'Hopital's Rule

$$\lim_{x\to 4} \frac{x^2-16}{x^2-5x+4} = \lim_{x\to 4} \frac{(x-4)(x+4)}{(x-4)(x-1)} = \lim_{x\to 4} \frac{x+4}{x-1} = \frac{8}{3}$$

$$\lim_{x\to 4} \frac{x^2-16}{x^2-5x+4} =$$

$$\lim_{x\to 0} \frac{2^x-1}{x} =$$

7 cases of indeterminate forms: $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $0 * \infty$, 0^0, 1^∞, ∞^0

L'Hopital's Rule Suppose that $f(x)$ and $g(x)$ are differential functions and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). If

$\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$

or

$\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$

then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

if the limit of the right side exists (or is ∞ or $-\infty$).

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case $\frac{0}{0}$ or $\frac{\infty}{\infty}$

Example: Evaluate these limits:

A) $\lim_{x\to 0} \frac{e^x+e^{-x}-2}{1-\cos(2x)} =$

B) $\lim\limits_{x \to 0} \dfrac{e^x + e^{-x}}{x^2} =$

C) $\lim\limits_{x \to \frac{\pi}{2}^-} \dfrac{4\tan(x)}{1 + \sec(x)} =$

case: $\infty - \infty$

Example: Evaluate these limits:

A) $\lim\limits_{x \to \frac{\pi}{2}^-} \sec(x) - \tan(x) =$

B) $\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right) =$

C) $\lim_{x\to 0^+} \ln(x) - \frac{1}{x} =$

case: $0 * \infty$

Example: Evaluate these limits:

A) $\lim_{x \to 1^+} \ln(x) \tan\left(\frac{\pi x}{2}\right)$

B) $\lim_{x \to \frac{\pi}{2}^-} (2x - \pi) \sec(x) =$

case: ∞^0, 1^∞, 0^0

Example: Evaluate these limits:

A) $\displaystyle\lim_{x\to 1^+} (2-x)^{\frac{4}{x-1}}$

B) $\displaystyle \lim_{x\to\infty} \left(1+\frac{3}{x}\right)^{2x} =$