Sections 4.1-4.3 Part 2: Increase, Decrease, Concavity, and Local Extrema

Definition: A **critical number (critical value)** is a number, c, in the domain of f such that $f'(c) = 0$ or $f'(c)$ DNE.

If f has a **local extrema** (local maxima or minima) at c then c is a critical value of $f(x)$.

Fermat's Theorem: If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

Example: Find the intervals where the function is increasing and the intervals where it is decreasing. Classify all critical values.

A) $y = x^3 + 3x^2 - 9x + 8$

B) $y = 3x^5 - 20x^3 + 20$

C) $y = \dfrac{x^2 + 1}{x}$

D) $y = (x^2 - 16)^{2/3}$

E) $y = x\ln(x)$

F) $y' = \dfrac{(x-4)^3(x+2)^2}{(x-1)}$ with the domain of y being all real numbers except $x = 1$.

Definition: $x = c$ is a possible inflection value (piv) provided that $x = c$ is in the domain of $f(x)$ and $f''(c) = 0$ or $f''(c)$ DNE.

Example: Find the intervals where the function is concave up and the intervals where it is concave down. Find the x-coordinate of the inflection points.

$y = x^5 - 5x^4 + 10x + 5$

Example: Find the values of a and b so that $f(x) = ax^2 - b\ln(x)$ will have an inflection point at $(1, 5)$

Example: The domain of the function $f(x)$ is all real numbers except $x = -5$. Use this information as well as f' and f'' to sketch a possible graph for $f(x)$.

$$f'(x) = \frac{-3x+7}{(x+5)^3} \qquad f''(x) = \frac{6(x-6)}{(x+5)^4}$$

Second Derivative Test: Suppose that f'' is continuous near the critical value c.

(a) If $f''(c) > 0$ then $f(x)$ has a ______________________________ at $x = c$.

(b) If $f''(c) < 0$ then $f(x)$ has a ______________________________ at $x = c$.

(c) If $f''(c) = 0$ then no conclusion can be made.

Example: Suppose that f has critical values of $x = 0$, $x = 2$, and $x = -2$. If $f''(x) = 60x^3 - 120x$, what conclusion can be drawn about the critical values?

Example: What conclusion can be made if you know that $g''(5) = 7$?