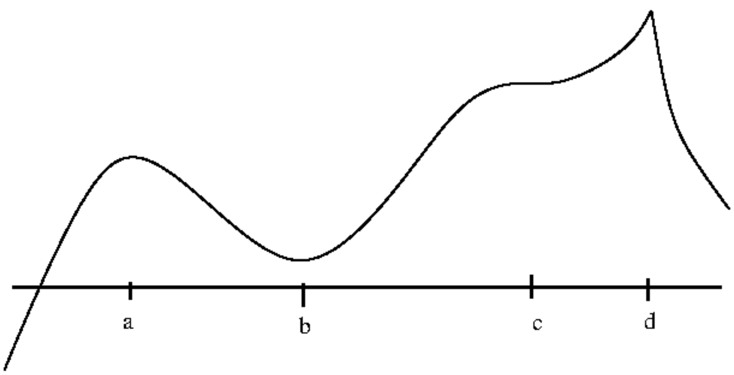
Section 4.1-4.3 Part 1: What Does f' and f'' say about f?

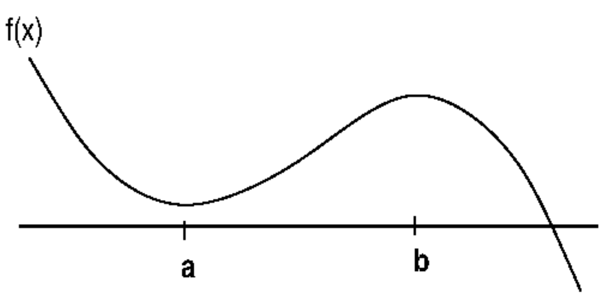
Definition: A function is said to be **increasing** on an interval if for a and b in the interval with $a < b$, then $f(a) < f(b)$. A function is said to be **decreasing** on an interval if for a and b in the interval with $a < b$, then $f(a) > f(b)$.



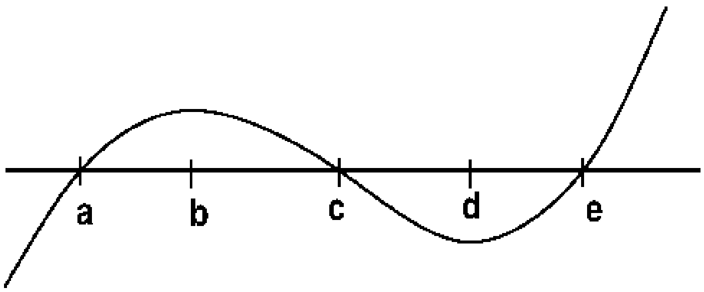
Definition: A **critical number (critical value)** is a number, c, in the domain of f such that

Definition: A function has a **local maximum (relative maximum)** at c if $f(c) \geq f(x)$ on an open interval that contains c, i.e. when x is near c. The value of the local maximum is $f(c)$.
Similarly, a function has a **local minimum (relative minimum)** at c if $f(c) \leq f(x)$ on an open interval that contains c. The value of the local minimum is $f(c)$.

Discuss the properties of the the derivate $f'(x)$ and how it relates to the properties of $f(x)$.



Example: Here is the graph of $f'(x)$.



A) Where is $f(x)$ increasing?

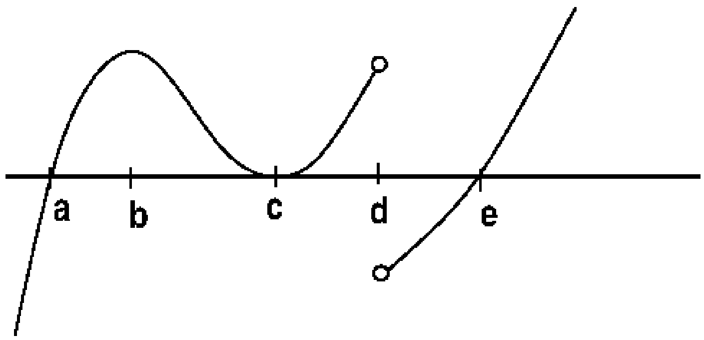
B) Where is $f(x)$ decreasing?

C) Where does $f(x)$ have a local minimum?

D) Where does $f(x)$ have a local maximum?

E) Sketch a possible graph of $f(x)$.

Example: Here is the graph of $f'(x)$. The domain of $f(x)$ is all real numbers.



A) Where is $f(x)$ increasing?

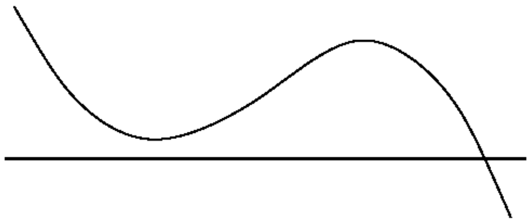
B) Where is $f(x)$ decreasing?

C) Where does $f(x)$ have a local minimum?

D) Where does $f(x)$ have a local maximum?

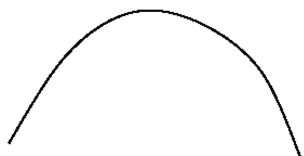
E) Sketch a possible graph of $f(x)$.

Example: Examine the concavity of the function $f(x)$.

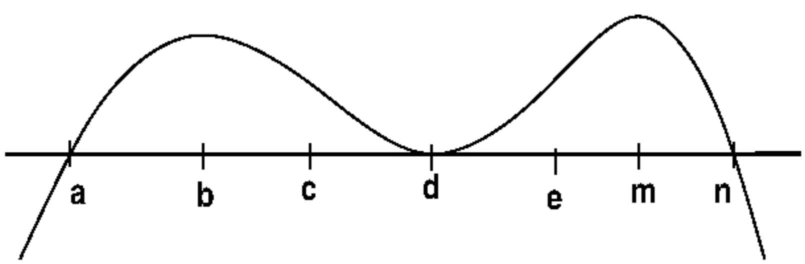


Definition: An **inflection point** is a point on the graph of $f(x)$ where $f(x)$ changes concavity.

Discuss the properties of the the derivate $f''(x)$ and how it relates to concavity of $f(x)$.



Example: Here is the graph of $f''(x)$.

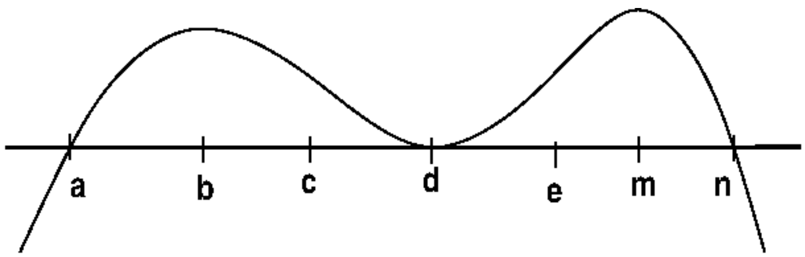


A) Where is $f(x)$ concave up?

B) Where is $f(x)$ concave down?

C) Find all $x-$values of the inflection points.

Example: Here is the graph of $f'(x)$.



A) Where is $f(x)$ concave up?

B) Where is $f(x)$ concave down?

C) Find all $x-$values of the inflection points.

Example: Sketch the graph of a function that meets these conditions.

Continuous and differentiable for all real numbers.
$f'(-1) = 0$ and $f'(5) = 0$

$f'(x) > 0$ on $(-1, 5)$, $(5, \infty)$
$f'(x) < 0$ on $(-\infty, -1)$

$f''(x) > 0$ on $(-\infty, 2)$, $(5, \infty)$
$f''(x) < 0$ on $(2, 5)$

Example: Sketch the graph of a function that meets these conditions.

$f'(1) = 0$, $f(0) = 1$, $\lim_{x\to\infty} f(x) = 3$

$f'(x) > 0$ on $(0, 1)$
$f'(x) < 0$ on $(-\infty, 0)$, $(1, \infty)$

$f''(x) < 0$ on $(0, 2)$
$f''(x) > 0$ on $(-\infty, 0)$, $(2, \infty)$