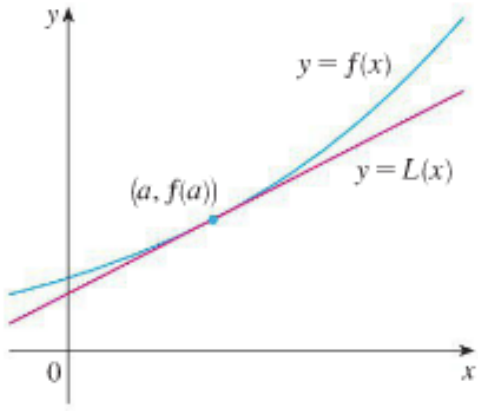
Section 3.10: Linear approximation and Differentials

Linear Approximation

Definition The formula $L(x) = f(a) + f'(a)(x-a)$ is called the **linear approximation** or **linearization** of $f(x)$ at $x = a$.



Example: Use $y = e^x$ to answer these questions.

A) Find the linearization at $a = 0$.

B) Use the linearization to approximate e^1 and $\frac{1}{e^{.25}}$

C) Find the values of x where the approximation is accurate to within 0.4.

Example: Find the linearization of $y = \cos(x)$ at $a = 60^o$. Use it to estimate $\cos(61^o)$ and $\cos(59^o)$.

Example: Use $y = \sqrt{x+7}$ to answer these questions.

A) Find the linearization at $a = 2$

B) Evaluate $\sqrt{9.06}$ and $\sqrt{11}$

C) Find the values of x where the approximation is accurate to within 0.5.

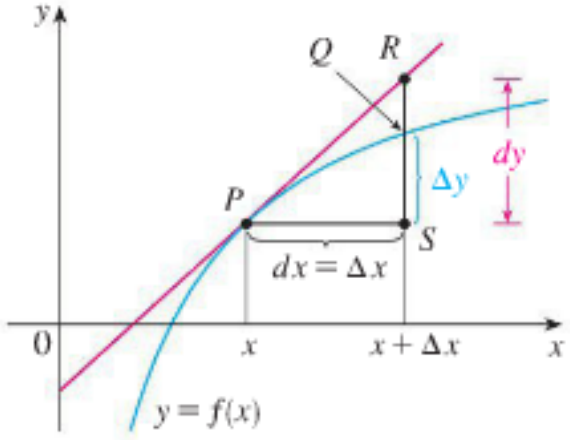
Definition let $y = f(x)$, where f is a differentiable function. Then the **differential** dx is an independent variable; that is dx can be given the value of any real number. The **differential** dy is then defined in terms of dx by the equation $dy = f'(x)dx$.

Example: Find dy and evaluate dy for the values of $x = 2$ and $dx = 0.3$.

$y = x^3 + 2x + 7$

Example: Find dy and evaluate dy for the values of $x = 1$ and $dx = 0.4$.

$y = \sqrt{x^2 + 3}$



Example: Use differentials to estimate $\sqrt[4]{16.1}$.

Example: The edge of a cube is measured to be 20 inches with a maximum error of 0.1 inches. What is the maximum error in the volume? What is the relative error? What is the percentage error?

Example: Use differentials to estimate the amount of paint needed to apply a coat of paint 0.05 cm thick to a hemispherical dome with radius of 25 m.