Section 2.8: Derivative

Definition: The **derivative of a function** f at a number a, denoted $f'(a)$, is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Example: Find the derivative of $f(x) = \dfrac{2}{x+5}$ at $a = 0$, $a = 2$, $a = 3$, $a = -5$.

Definition of the Derivative: The derivative of a function $f(x)$, denoted $f'(x)$ is

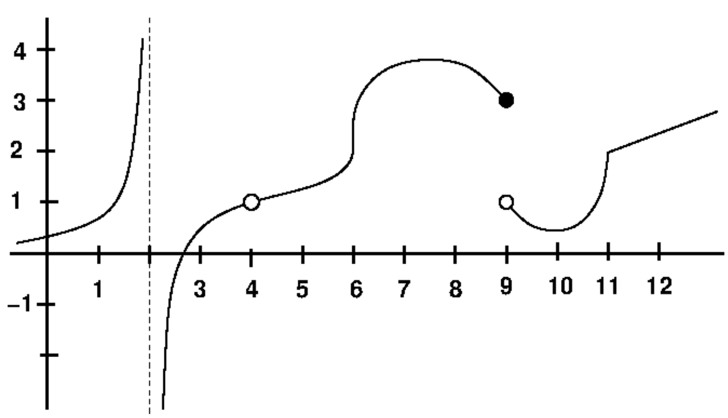
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Other common notations for the derivative are f', $\frac{dy}{dx}$, and $\frac{d}{dx}f(x)$

Note: Once you have the function $f'(x)$, also called the **first derivative**, you can redo the derivative process with that function and compute the **second derivative** (notation: $f''(x)$, y'', $\frac{d^2y}{dx^2}$...).

Example: For the function $f(x) = \frac{2}{x+5}$, find the equation of the tangent line at $x = 3$.

Example: Here is the graph of $f(x)$. Where does the derivative not exist?



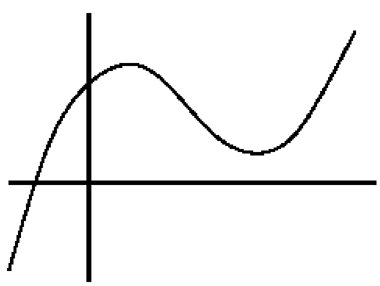
Definition: $f(x)$ is said to be **differentiable** at $x = a$ provided that $f'(a)$ exists. $f(x)$ is differentiable on an open interval (a, b) provided it is differentiable at every number in the interval.

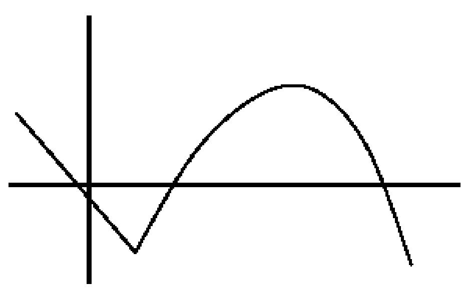
Theorem: If f is differentiable at a, then f is continuous at a.

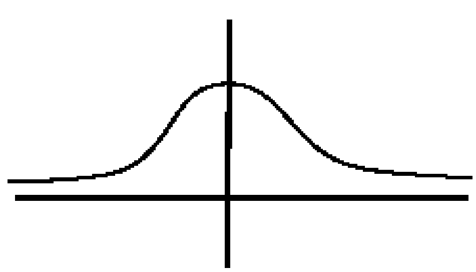
Example: Sketch the graph of $f(x)$ and use this graph to find $f'(x)$. Give the values where $f(x)$ is not continuous and where it is not differentiable.

$f(x) = |2x - 4|$

Example: Sketch the graph of the derivative for these graphs.







Example: Use the definition of the derivative to find $g'(x)$ for $g(x) = 3x^2 + 2x + 7$

Example: Use the definition of the derivative to find $g'(x)$ for $g(x) = \sqrt{3x+5}$.