Section 2.3: Calculating Limits Using Limit Laws

Limit Laws Suppose that c is a constant and the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exists. Then

1) $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

2) $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

3) $\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x)$

4) $\lim_{x\to a} f(x) * g(x) = \lim_{x\to a} f(x) * \lim_{x\to a} g(x)$

5) $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$

6) $\lim_{x\to a} [f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$, where n is a positive integer

7) $\lim_{x\to a} c = c$

8) $\lim_{x\to a} x = a$

9) $\lim_{x\to a} x^n = a^n$ where n is a positive integer

10) $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer and if n is even, then we assume that $a > 0$

11) $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$, where n is a positive integer.

Example: Suppose $\lim_{x\to a} f(x) = 5$ and $\lim_{x\to a} g(x) = 2$, compute

$\lim_{x\to a} \dfrac{2f(x) - 3g(x)}{(f(x))^2} =$

Example: Compute the following limits.

A) $\lim_{x\to 2} 4x^3 + 5 =$

B) $\lim_{x \to -3} \frac{x^2 - 9}{x + 5}$

C) $\lim_{x \to -3} \frac{x^2 - 9}{x^2 + x - 6}$

D) $\lim_{h \to 0} \frac{(4 + h)^2 - 16}{h}$

Example: Use the function $f(x)$ to answer these questions. $f(x) = \begin{cases} x^3 - 2x + 4 & \text{if } x \leq 2 \\ 3x - 2 & \text{if } x > 2 \end{cases}$

A) $\lim\limits_{x \to 1} f(x)$

B) $\lim\limits_{x \to 2} f(x)$

Example: Evaluate these limits.

A) $\lim\limits_{x \to 3} \dfrac{|x-3|}{x-3}$

B) $\lim_{x \to 2} \dfrac{x^{-1} - .5}{x - 2}$

C) $\lim_{x \to 0} \dfrac{\sqrt{3 - x} - \sqrt{3}}{x}$

Squeeze Theorem If $f(x) \leq g(x) \leq h(x)$ for all x in an open interval about a (except possibly at a) and $\lim_{x \to a} f(x) = L = \lim_{x \to a} h(x)$ then $\lim_{x \to a} g(x) = L$

Example: Compute $\lim_{x \to 0^+} \sqrt{x} e^{sin(3/x)}$