

## Section 2.2: The Limit of a Function

A limit is way to discuss how the values of a function(y-values) are behaving when  $x$  gets close to the number  $a$ . There are three forms to the limit.

$$\lim_{x \rightarrow a^-} f(x)$$

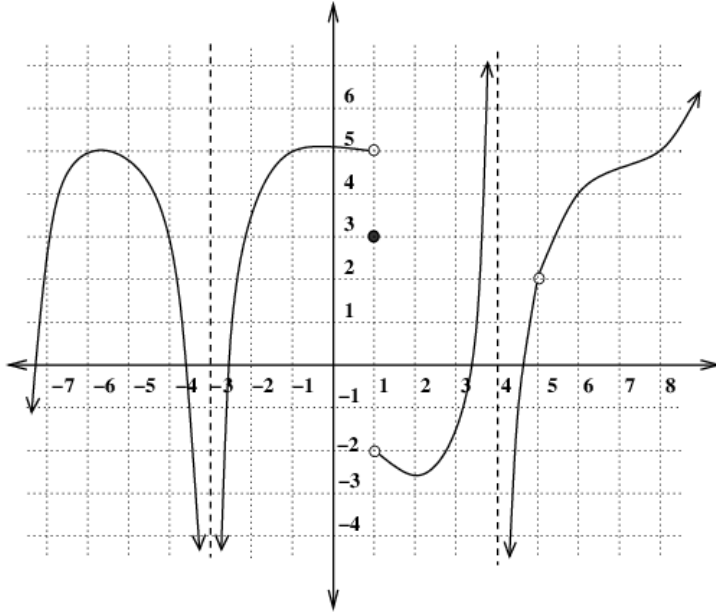
$$\lim_{x \rightarrow a^+} f(x)$$

$$\lim_{x \rightarrow a} f(x)$$

We write  $\lim_{x \rightarrow a^-} f(x) = L$  and say "the limit of  $f(x)$  as  $x$  approaches  $a$  from the left, equals  $L$ "

### Evaluating Limits Graphically

Example: Use the graph to answer the following questions.



$$\lim_{x \rightarrow -1^-} f(x) =$$

$$\lim_{x \rightarrow 1^-} f(x) =$$

$$\lim_{x \rightarrow 5^-} f(x) =$$

$$\lim_{x \rightarrow -1^+} f(x) =$$

$$\lim_{x \rightarrow 1^+} f(x) =$$

$$\lim_{x \rightarrow 5^+} f(x) =$$

$$\lim_{x \rightarrow -1} f(x) =$$

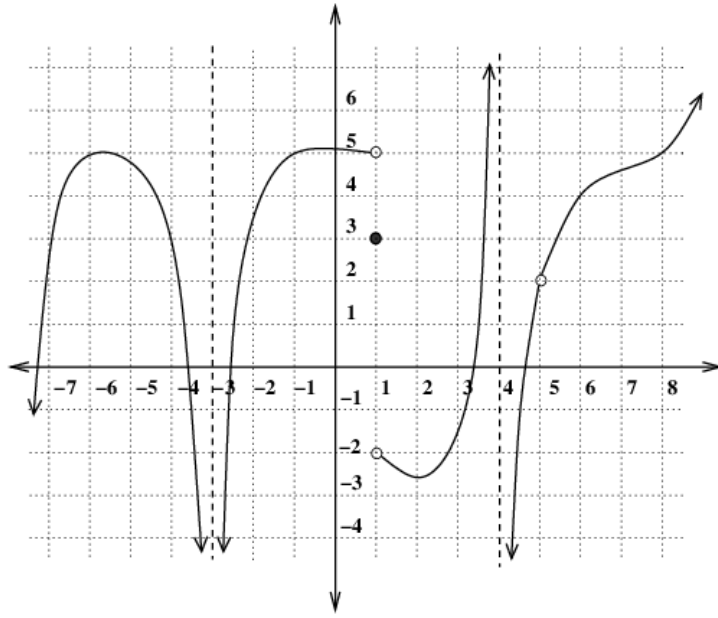
$$\lim_{x \rightarrow 1} f(x) =$$

$$\lim_{x \rightarrow 5} f(x) =$$

$$f(1) =$$

$$f(5) =$$

Example: Use the graph to answer the following questions.



$$\lim_{x \rightarrow -3^-} f(x) =$$

$$\lim_{x \rightarrow 4^-} f(x) =$$

$$\lim_{x \rightarrow \infty} f(x) =$$

$$\lim_{x \rightarrow -3^+} f(x) =$$

$$\lim_{x \rightarrow 4^+} f(x) =$$

$$\lim_{x \rightarrow -\infty} f(x) =$$

$$\lim_{x \rightarrow -3} f(x) =$$

$$\lim_{x \rightarrow 4} f(x) =$$

**Definition:**  $x = a$  is said to be a **vertical asymptote** of the function  $f(x)$  provided that at least one of the following statements is true:

$$\lim_{x \rightarrow a^-} f(x) = \infty$$

$$\lim_{x \rightarrow a^+} f(x) = \infty$$

$$\lim_{x \rightarrow a^-} f(x) = -\infty$$

$$\lim_{x \rightarrow a^+} f(x) = -\infty$$

**Evaluating Limits with Tables**

Example: Compute the limit.

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 16} - 4}{x^2} =$$

x	$f(x)$
1	0.1231056
0.5	0.124515
0.1	0.1249804
0.05	0.1249951
0.001	0.1249998

x	$f(x)$
-1	0.1231056
-0.5	0.124515
-0.1	0.1249804
-0.05	0.1249951
-0.001	0.1249998

Example: Compute the limit.

$$\lim_{x \rightarrow 0} \sin\left(\frac{\pi}{x}\right) =$$

Example: Evaluate these limits.

$$\text{A) } \lim_{x \rightarrow 4^+} \frac{1}{x-4} =$$

$$\text{B) } \lim_{x \rightarrow 0} \frac{1}{x^2} =$$

$$\text{C) } \lim_{x \rightarrow 0} \frac{1}{x^3} =$$