Appendix K.1: Derivatives of Vector Functions

Definition: $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ is a vector function, then the derivative of $\mathbf{r}(t)$ is given by

$$\mathbf{r}'(t) = \left\langle x'(t), y'(t) \right\rangle = x'(t)\mathbf{i} + y'(t)\mathbf{j}$$

provided both $x'(t)$ and $y'(t)$ exist.

When $\mathbf{r}(t)$ represents a position function, then $\mathbf{r}'(t)$ is velocity and $|\mathbf{r}'(t)|$ is speed.

Example: Assume that $\mathbf{r}(t)$ is a position function for an object. Find the velocity vector(s) and the speed at the point $(3, 0)$ when

$\mathbf{r}(t) = \left\langle t^2 - 6t + 8, t^4 - 26t^2 + 25 \right\rangle$

Example: Find the derivative of $\mathbf{r}(t) = \left\langle \dfrac{t}{\tan(t)}, \cos(3t^2) \right\rangle$.

Example: For the vector function, $\mathbf{r}(t) = \left\langle 10t^2, 5t^3 + 7 \right\rangle$, find a tangent vector of unit length when $t = 2$.