Math 152 Week in Review: Section 10.3, 10.4

1. Give two polar representations for the point $(-5\sqrt{3}, 5)$. One with $r > 0$ and one with $r < 0$.

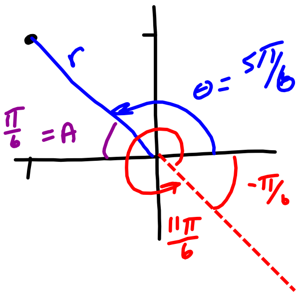
$x \quad y$

$r^2 = x^2 + y^2$

$\tan\theta = \frac{y}{x}$

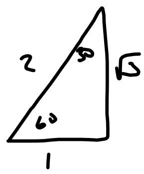
$x = r\cos\theta$

$y = r\sin\theta$



$r^2 = x^2 + y^2$
$= (-5\sqrt{3})^2 + 5^2$
$= 25(3) + 25$
$= 75 + 25$
$r^2 = 100$
$r = 10$

$\tan\theta = \frac{-5}{5\sqrt{3}} = \frac{-1}{\sqrt{3}}$



$\tan A = \frac{y}{x}$

$\tan A = \frac{5}{5\sqrt{3}}$

$A = \arctan\left(\frac{1}{\sqrt{3}}\right)$

$A = 30^\circ = \frac{\pi}{6}$

$\boxed{\begin{array}{ll} r > 0 & r < 0 \\ \left(10, \frac{5\pi}{6}\right) & \left(-10, -\frac{\pi}{6}\right) \\ \left(10, \frac{5\pi}{6} + 2\pi\right) & \left(-10, \frac{11\pi}{6}\right) \end{array}}$

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2. Write a Cartesian equation for the polar curve $r = -8 \sin\theta$

$r^2 = x^2 + y^2$

$x = r\cos\theta$

$y = r\sin\theta$

$r^2 = -8 r \sin\theta$

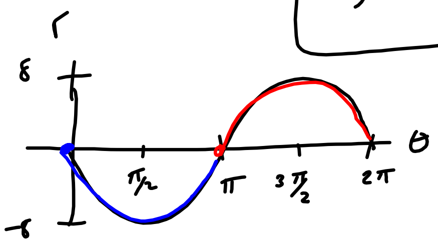
$x^2 + y^2 = -8y$

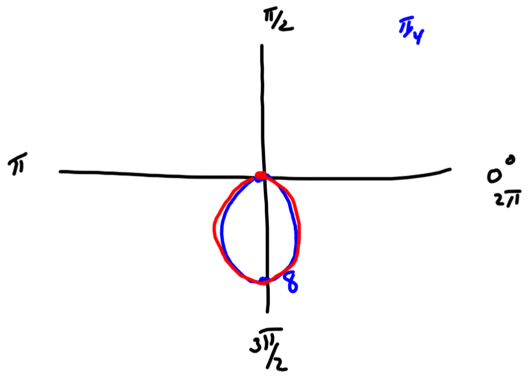
$x^2 + y^2 + 8y = 0$

$x^2 + y^2 + 8y + 4^2 = 4^2$

$x^2 + (y+4)^2 = 16$

center $(0, -4)$





3. Write a Cartesian equation for the polar curve $r^2 \sin(2\theta) = 1$.

$r^2 \cdot 2 \sin\theta \cos\theta = 1$

$2 r \sin\theta \, r\cos\theta = 1$

$2y\,x = 1$

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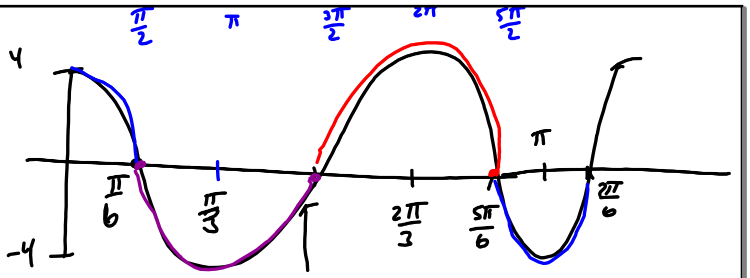
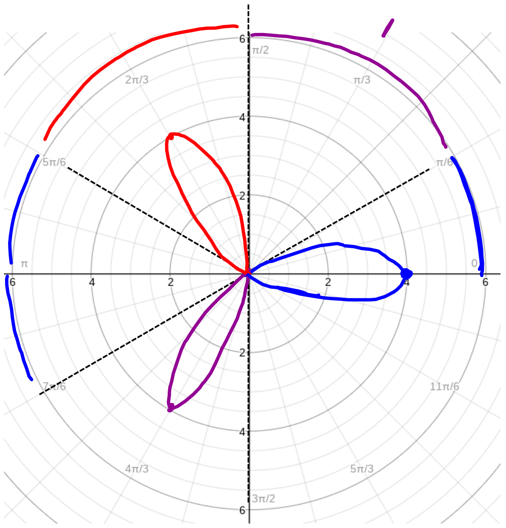
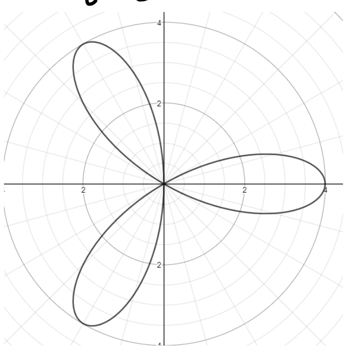
$3\theta = \frac{\pi}{2}$

4. Graph $r = 4\cos(3\theta)$



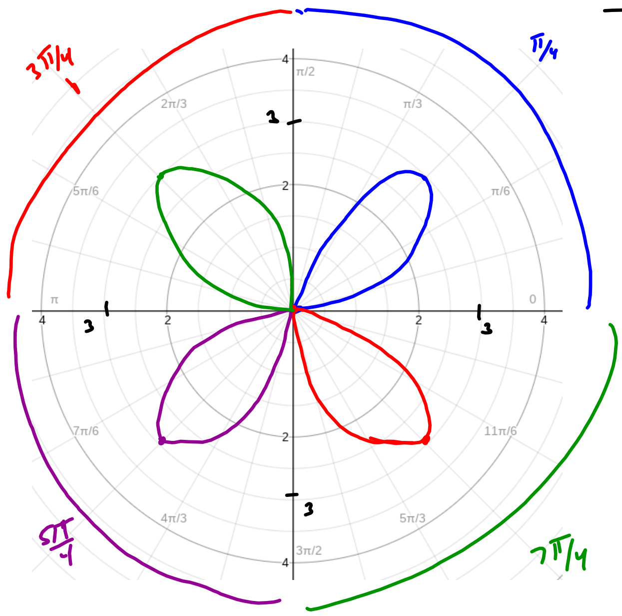


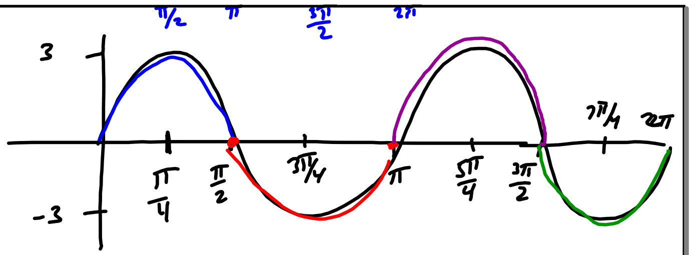
$\frac{3\pi}{6} = \frac{\pi}{2}$

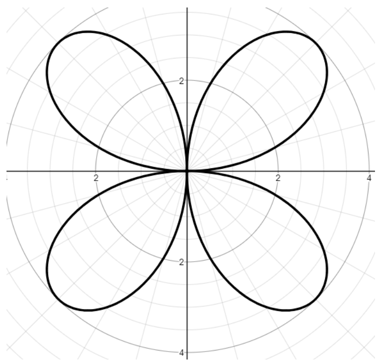


$2\theta = \frac{\pi}{2}$

5. Graph $r = 3\sin(2\theta)$







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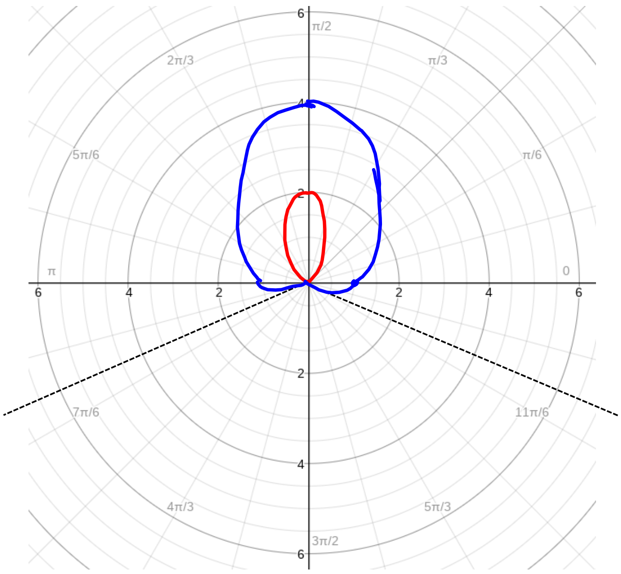
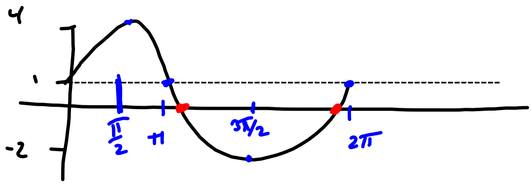
6. Graph $r = 1 + 3\sin\theta$

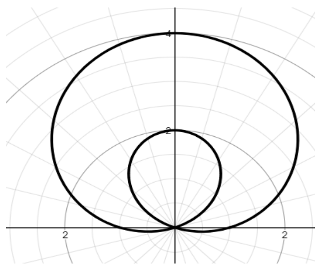




$0 = 1 + 3\sin\theta$

$3\sin\theta = -1$

$\sin\theta = \frac{-1}{3}$



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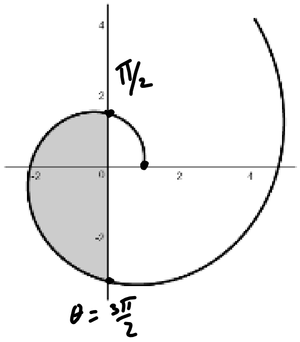
7. Find the area shaded in the region below for the curve $r = e^{\theta/4}$.

$$\text{Area} = \int_a^b \frac{1}{2} r^2 \, d\theta$$



$$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$$

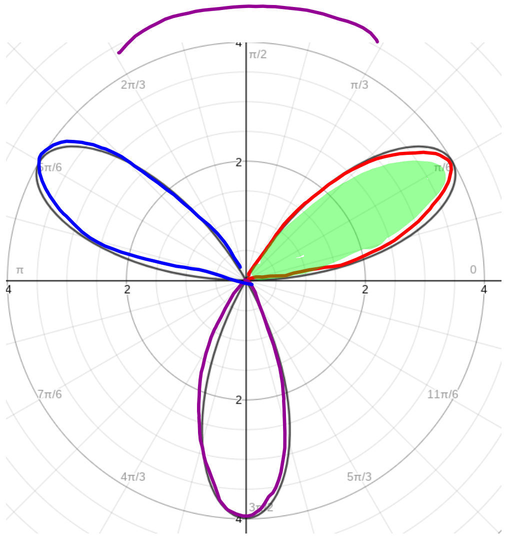
$$A = \int_{\pi/2}^{3\pi/2} \frac{1}{2} \left(e^{\theta/4}\right)^2 \, d\theta = \int_{\frac{\pi}{2}}^{3\pi/2} \frac{1}{2} e^{\theta/2} \, d\theta$$

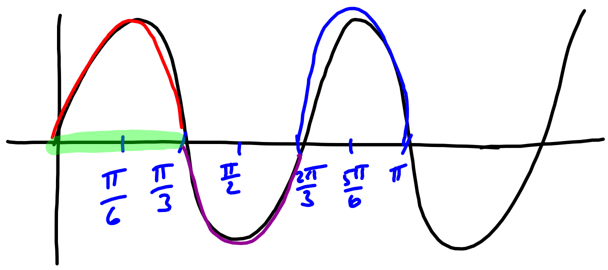
$$= \frac{1}{2} \cdot \frac{1}{\frac{1}{2}} e^{\theta/2} \Big|_{\pi/2}^{3\pi/2} = e^{\frac{1}{2} \cdot \frac{3\pi}{2}} - e^{\frac{1}{2} \cdot \frac{\pi}{2}}$$

$$= \underline{e^{3\pi/4} - e^{\pi/4}}$$

$\frac{\pi}{2}$ π $\frac{3\pi}{2}$ 2π $\frac{5\pi}{2}$

8. Find the area of inside one petal of the polar curve $r = 4\sin(3\theta)$.





$0 \le \theta \le \frac{\pi}{3}$

$$\text{Area} = \int_0^{\pi/3} \frac{1}{2}\left(4\sin(3\theta)\right)^2 \, d\theta$$

$$= \int_0^{\pi/3} \frac{1}{2}\, 16 \sin^2(3\theta)\, d\theta = \int_0^{\pi/3} 8\sin^2(3\theta)\, d\theta$$

$\cos^2\theta = \frac{1}{2}\left(1 + \cos 2\theta\right)$

$\sin^2\theta = \frac{1}{2}\left(1 - \cos 2\theta\right)$

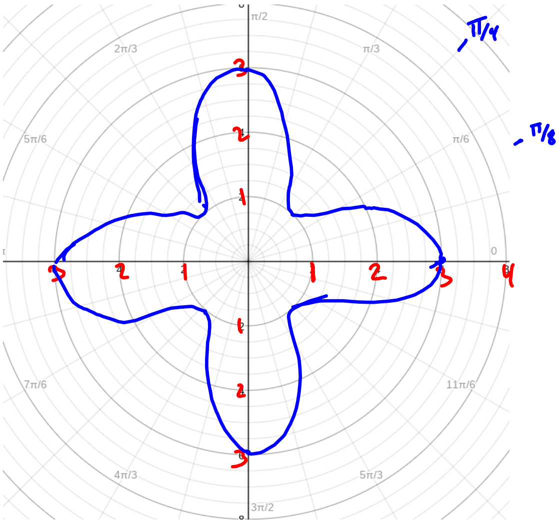
$$= \int_0^{\pi/3} 8 \cdot \frac{1}{2}\left(1 - \cos(6\theta)\right) d\theta$$

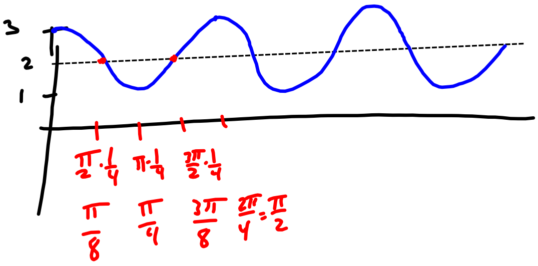
$$= 4\left(\theta - \frac{1}{6}\sin(6\theta)\right)\Big|_0^{\pi/3}$$

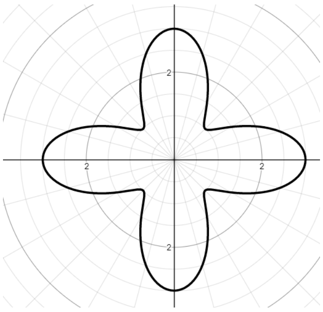
$$= 4\left[\frac{\pi}{3} - \frac{1}{6}\sin(2\pi) - \left(0 - \frac{1}{6}\sin(0)\right)\right]$$

$$= 4\left[\frac{\pi}{3} - 0 - (0 - 0)\right] = \frac{4\pi}{3}$$

9. Find the area inside the polar curve $r = 2 + \cos(4\theta)$.







$0 \le \theta \le 2\pi$

$$\text{Area} = \int_0^{2\pi} \frac{1}{2}\left(2+\cos(4\theta)\right)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} 4 + 4\cos 4\theta + \cos^2(4\theta)\, d\theta$$

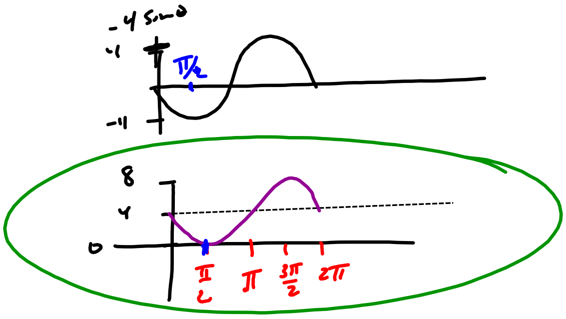
$$= \frac{1}{2}\int_0^{2\pi} 4 + 4\cos 4\theta + \frac{1}{2}\left[1+\cos 8\theta\right] d\theta$$

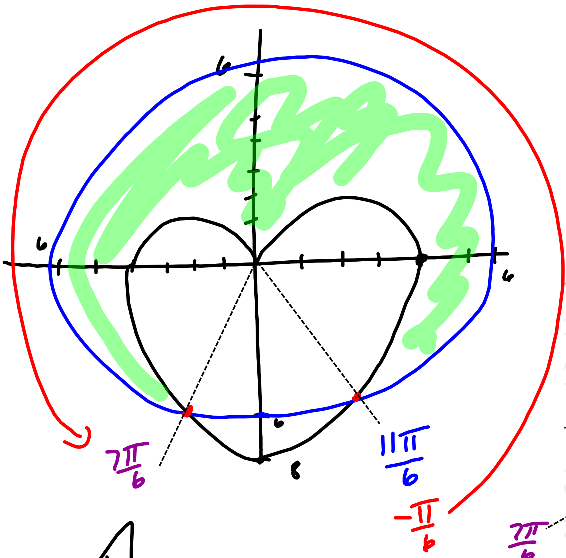
$$= \frac{1}{2}\left(4\theta + \frac{4}{4}\sin 4\theta + \frac{1}{2}\left[\theta + \frac{1}{8}\sin 8\theta\right]\right)\Bigg|_0^{2\pi}$$

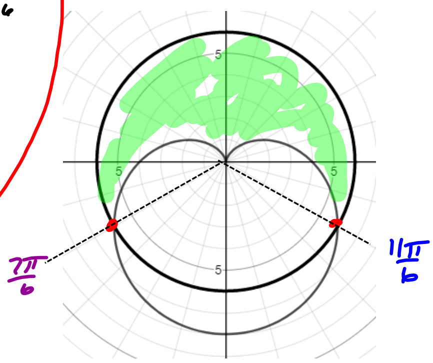
$$= \frac{1}{2}\left(8\pi + 0 + \frac{1}{2}\left[2\pi + 0\right] - 0\right)$$

$$= \frac{1}{2}\left(8\pi + \pi\right) = \frac{9\pi}{2}$$

10. Set up the integral for the area inside the circle $r = 6$ and outside the cardioid $r = 4 - 4\sin\theta$.







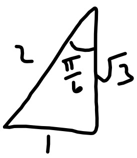
$4 - 4\sin\theta = 6$

$-4\sin\theta = 2$

$\sin\theta = -\frac{2}{4} = -\frac{1}{2}$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}$



$$\text{Area} = \int_{-\pi/6}^{7\pi/6} \frac{1}{2}\left[6^2 - (4 - 4\sin\theta)^2\right] d\theta$$

$$\int_{\pi/2}^{7\pi/6} 6^2 - (4 - 4\sin\theta)^2 \, d\theta$$

$r' = 3e^{3\theta}$

11. Find the length of the polar curve. $r = e^{3\theta}$, $0 \leq \theta \leq 2\pi$

$$\int_a^b ds$$

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$ds = \sqrt{(x')^2 + (y')^2}\, dt$$

$$\text{Arclength} = \int_0^{2\pi} \sqrt{(e^{3\theta})^2 + (3e^{3\theta})^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{e^{6\theta} + 9e^{6\theta}}\, d\theta = \int_0^{2\pi} \sqrt{10e^{6\theta}}\, d\theta$$

$$= \int_0^{2\pi} e^{3\theta}\sqrt{10}\, d\theta = \frac{\sqrt{10}}{3} e^{3\theta}\Big|_0^{2\pi}$$

$$= \frac{\sqrt{10}}{3} e^{6\pi} - \frac{\sqrt{10}}{3} e^0 = \frac{\sqrt{10}}{3}\left(e^{6\pi} - 1\right)$$

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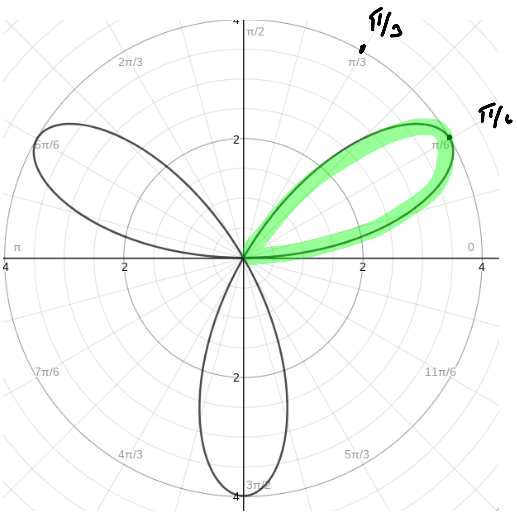
$r' = 4 \cdot \cos(3\theta) \cdot 3 = 12\cos(3\theta)$

12. Set up an integral for the arc length of one petal $r = 4\sin(3\theta)$.



$$\int_0^{\pi/3} \sqrt{(4\sin(3\theta))^2 + (12\cos(3\theta))^2}\, d\theta$$

$$= \int_0^{\pi/3} \sqrt{16\sin^2(3\theta) + 144\cos^2(3\theta)}\, d\theta$$