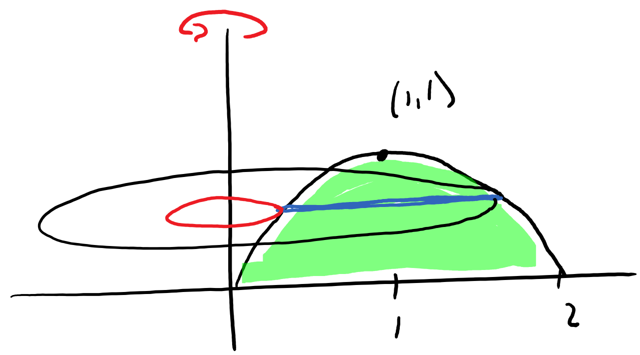
Example: Find the volume of the solid obtained by rotating the region bounded by the given curves around the y-axis.

$y = 2x - x^2 = x(2-x)$
x−axis

dy Integral.



solve for x

by completing the square

$$x = 1 \pm \sqrt{1-y}$$

$$x = \underbrace{1 - \sqrt{1-y}}_{r_i} \qquad x = 1 + \sqrt{1-y} = r_o$$

$$V = \int_0^1 \pi\left[\left(1+\sqrt{1-y}\right)^2 - \left(1-\sqrt{1-y}\right)^2\right] dy$$

Page 1:method 2

disk/washer $\to$ slice is perp. to axis of Rotation

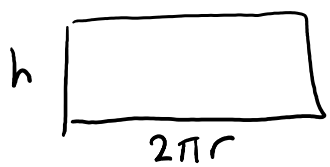
Shells $\to$ Slice is parallel to axis of Rotation

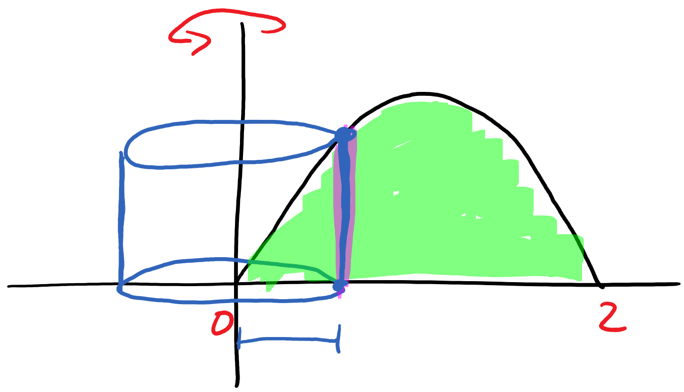
Example: Find the volume of the solid obtained by rotating the region bounded by the given curves around the y-axis.

$y = 2x - x^2 = x(2-x)$

$x-$axis

shell (outside of a can)





$$V = \int_a^b 2\pi r h \, dx$$

$r = x$

$h = y = 2x - x^2$

$$V = \int_0^2 2\pi x(2x - x^2)\, dx = \dots = \frac{8\pi}{3}$$

nesting dolls

shells. dy Integral.

Example: Set up the integral(s) that would give the volume of the solid obtained by rotating the region bounded by the given curves around x-axis.

$y = x^2$ → $x = \pm\sqrt{y} \Rightarrow x = +\sqrt{y}$

$y^2 = 8x$ → $x = \frac{1}{8}y^2$

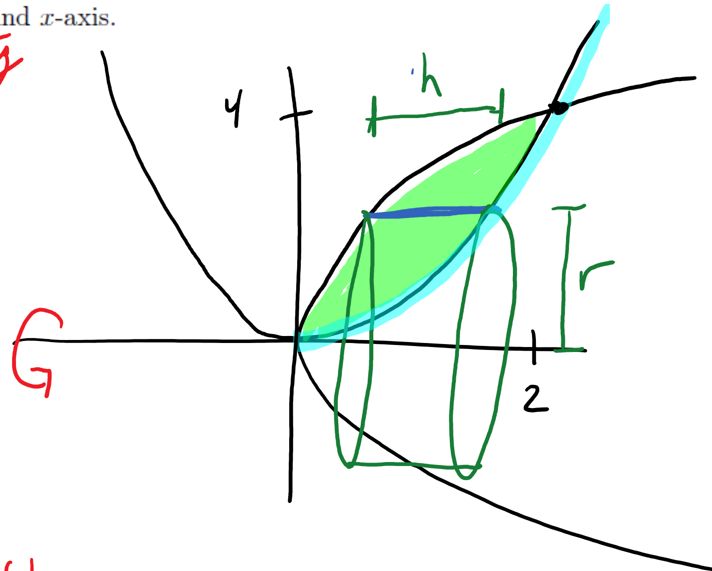
$(x^2)^2 = 8x$

$x^4 = 8x$

$x^4 - 8x = 0$

$x(x^3 - 8) = 0$

$x = 0 \quad x = 2$



$r = y$

$h =$ Right − Left

$\sqrt{y} - \frac{1}{8}y^2$

$$V = \int_0^4 2\pi\, y \left(\sqrt{y} - \frac{1}{8}y^2\right) dy = \cdots = \frac{48\pi}{5}$$

---

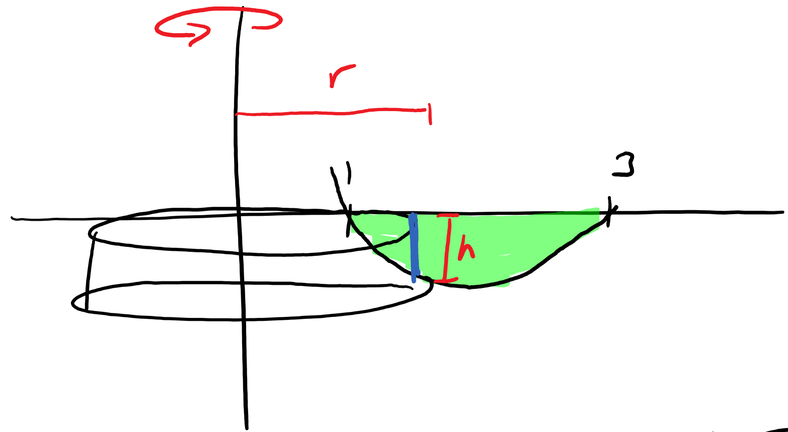
washer

$$V = \int_0^2 \pi\left[(\sqrt{8x})^2 - (x^2)^2\right] dx = \int_0^2 \pi\left(8x - x^4\right) dx$$

Shell
dx Integral

Example: Set up the integral(s) that would give the volume of the solid obtained by rotating the region bounded by the given curves around y-axis.

$y = x^2 - 4x + 3 = (x-3)(x-1)$
x−axis



$r = x$

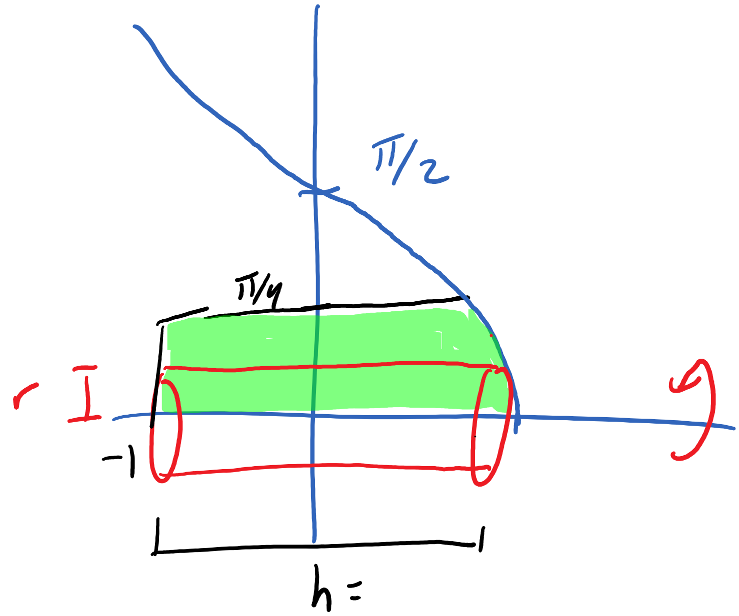
$h = 0 - y$
$= 0 - (x^2 - 4x + 3)$
$= -x^2 + 4x - 3$

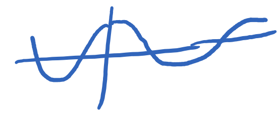
$$V = \int_1^3 2\pi x(-x^2 + 4x - 3)\,dx = \frac{16\pi}{3}$$

Example: Set up the integral(s) that would give the volume of the solid obtained by rotating the region bounded by the given curves around x-axis on the interval $y = 0$ to $y = \frac{\pi}{4}$

$x = \cos(y)$
$x = -1$

dy Integral.





$r = y$

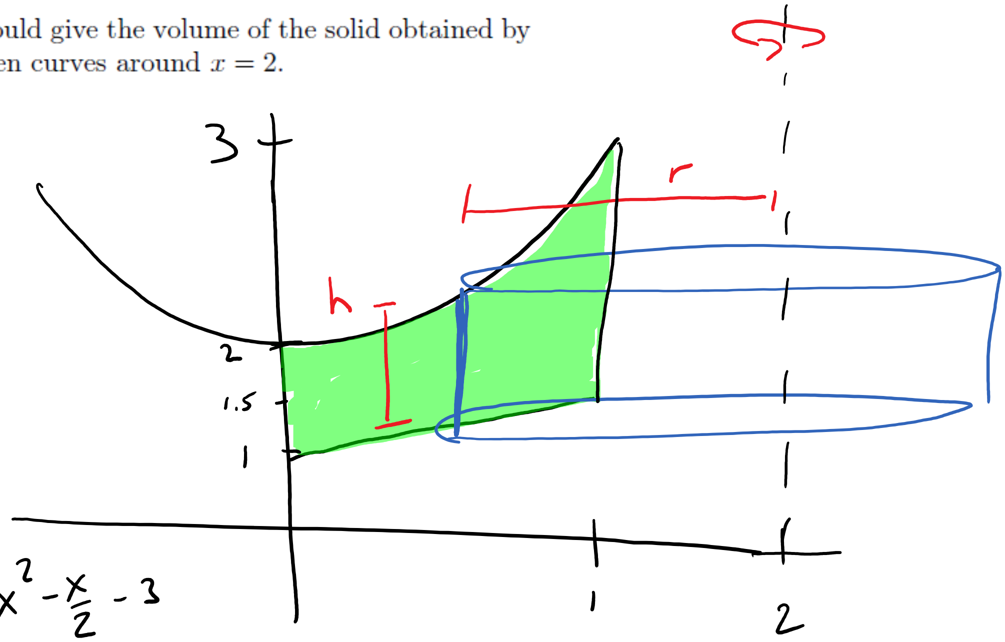
$h = \cos(y) - (-1)$

$= \cos(y) + 1$

$$V = \int_0^{\pi/4} 2\pi y \left(\cos(y) + 1\right) dy$$

Example: Set up the integral(s) that would give the volume of the solid obtained by rotating the region bounded by the given curves around $x = 2$.

$y = x^2 + 2$
$2y - x = 2 \rightarrow y = \frac{x}{2} + 1$
$x = 0$
$x = 1$



Shells
dx Integral

$r = 2 - x$

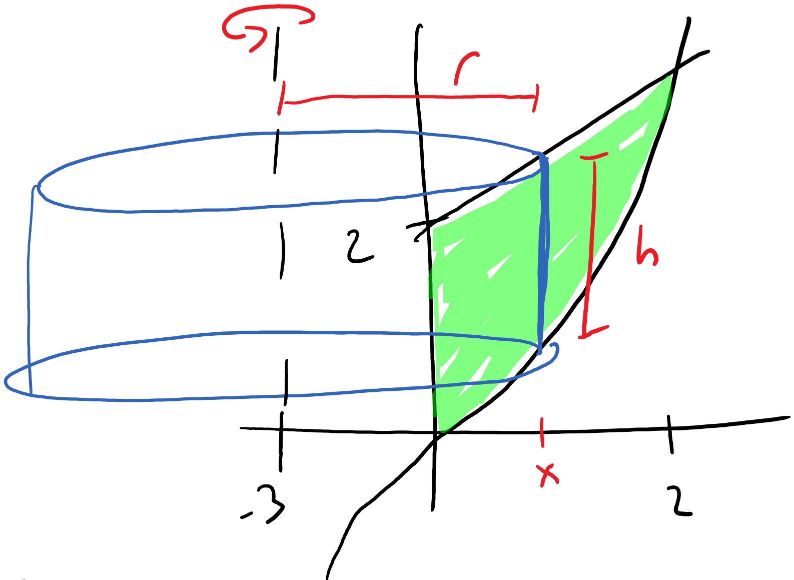
$h = x^2 + 2 - \left(\frac{x}{2} + 1\right) = x^2 - \frac{x}{2} - 3$

$$V = \int_0^1 2\pi(2-x)\left(x^2 - \frac{x}{2} - 3\right)dx = \cdots = \frac{19\pi}{6}$$

dx Integral

Example: Set up the integral(s) that would give the volume of the solid obtained by rotating the region bounded by the given curves around $x = -3$.

$y = x^3$
$y = 2x + 4$
$x = 0$



$r = x - (-3)$
$r = x + 3$

$h = 2x + 4 - x^3$

$$V = \int_0^2 2\pi (x+3)(2x+4-x^3)\,dx$$