

Section 10.1: Curves Defined by Parametric Equations

Example: Use the parametric function $x(t) = t^2 + 3t$, $y(t) = 2t + 5$ to answer the following.

A) Is the point $(10, 8)$ on the graph? Justify your answer.

NO!

$$x = t^2 + 3t$$

$$= (1.5)^2 + 3(1.5)$$

$$= 2.25 + 4.5$$

$$= 6.75 \neq 10$$

$$y = 2t + 5$$

$$8 = 2t + 5$$

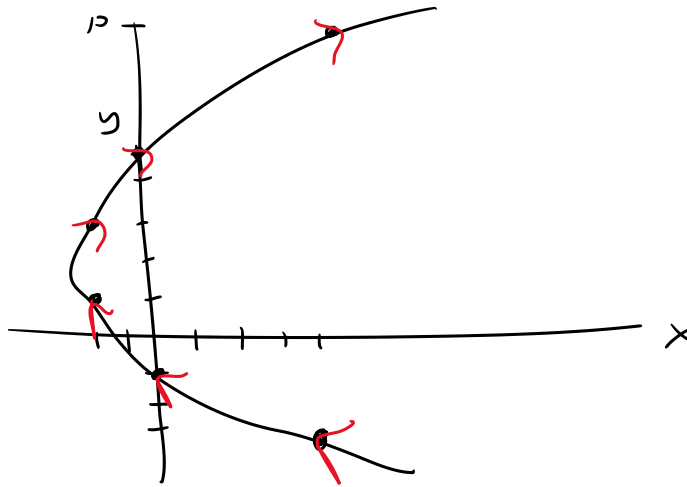
$$3 = 2t$$

$$1.5 = \frac{3}{2} = t$$

B) Sketch the graph of the curve.

values of t

t	x	y
-4	4	-3
-3	0	-1
-2	-2	1
-1	-2	3
0	0	5
1	4	7
2	10	9



C) Find the Cartesian equation of the parametric function.

$$x = t^2 + 3t$$

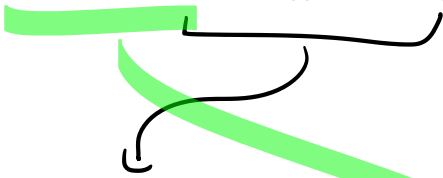
$$y = 2t + 5$$

$$y - 5 = 2t$$

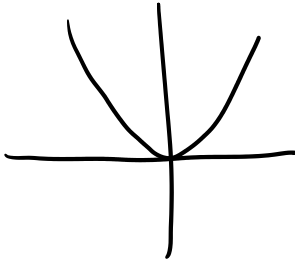
$$\frac{y - 5}{2} = t$$

$$x = \left(\frac{y-5}{2}\right)^2 + 3\left(\frac{y-5}{2}\right)$$

Example: Sketch the curve $x = \cos(t)$, $y = \cos^2(t)$.



$$y = x^2$$

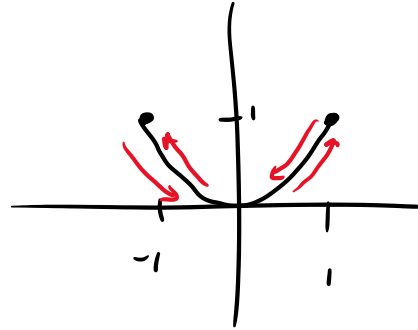


$$x = \cos t$$

$$-1 \leq x \leq 1$$

$$y = \cos^2 t$$

$$0 \leq y \leq 1$$



path of a circle of radius r .

$x = r \cos \theta$	$x = r \sin \theta$
$y = r \sin \theta$	$y = r \cos \theta$

Example: Sketch the graph of these parametric curves.

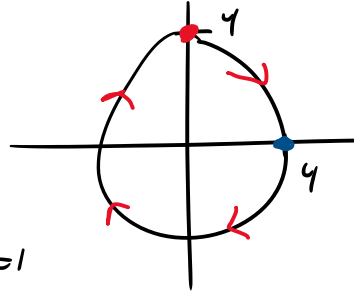
A) $x = 4 \sin(t)$, $y = 4 \cos(t)$

$$\frac{x}{4} = \sin t \quad \frac{y}{4} = \cos t$$

$$\sin^2(t) + \cos^2(t) = 1$$

$$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = 1 \quad \rightarrow \quad \frac{x^2}{16} + \frac{y^2}{16} = 1$$

$$x^2 + y^2 = 16$$



$$\theta = 0$$

$$x = 0 \quad y = 4$$

$$\theta = \frac{\pi}{2}$$

$$x = 4 \quad y = 0$$

B) $x = 4 \cos(t)$, $y = 4 \sin(t)$

