Sections 5.5: The Substitution Rule

Knowing $\underline{f(x) = (x^3+2)^4}$ and $\underbrace{f'(x) = 4(x^3+2)^3 * 3x^2 = 12x^2(x^3+2)^3}$

Compute $\int 12x^2(x^3+2)^3 dx =$ $(x^3+2)^4 + C$

Example: Compute.

$\frac{1}{9}\int 9 \; 2x(x^2+5)^8 \; dx =$ $\frac{1}{9}(x^2+5)^9 + C$

The substitution Rule If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$\int f(g(x))g'(x)\, dx = \int f(u)\, du$

Example: Compute the following.

A) $\int \cos(5x)\, dx = \quad = \int \cos(u) \cdot \frac{1}{5}\, du$

$u = 5x$

$du = 5\, dx$

$\frac{1}{5}\, du = dx$

$= \int \frac{1}{5} \cos(u)\, du = \frac{1}{5} \sin(u) + C$

$= \frac{1}{5} \sin(5x) + C$

$\int \cos(7x)\, dx = \frac{1}{7} \sin(7x) + C$

$\int \sin(3x)\, dx = -\frac{1}{3} \cos(3x) + C$

B) $\int 2x^3(x^4+7)^5\,dx = \int 2x^3 u^5 \cdot \frac{1}{4x^3}\,du = \int \frac{1}{2} u^5\,du$

$u = x^4 + 7$

$du = 4x^3\,dx$

$\frac{1}{4x^3}\,du = dx$

$= \frac{1}{2}\frac{u^6}{6} + C$

$= \frac{1}{12} u^6 + C$

$= \frac{1}{12}(x^4+7)^6 + C$

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C) $\int \frac{12x^3+9}{(x^4+3x)^5}\,dx = \int \frac{12x^3+9}{u^5}\cdot\frac{1}{4x^3+3}\,du = \int \frac{3\cancel{(4x^3+3)}}{u^5}\cdot\frac{1}{\cancel{(4x^3+3)}}\,du$

$u = x^4+3x$

$du = (4x^3+3)\,dx$

$\frac{1}{4x^3+3}\,du = dx$

$$= \int \frac{3}{u^5}\,du = \int 3u^{-5}\,du$$

$$= \frac{3u^{-4}}{-4} + C = \frac{-3}{4u^4} + C$$

$$= \boxed{\frac{-3}{4(x^4+3x)^4} + C}$$

D) $\int \frac{e^{2+\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{e^u}{\sqrt{x}}\,2\sqrt{x}\,du = \int 2e^u\,du$

$= 2e^u + C$

$= 2e^{2+\sqrt{x}} + C$

$u = 2+\sqrt{x}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,du = dx$

E) $\int x(x-8)^8\,dx = \int x\,u^8\,du = \int (u+8)u^8\,du$

$u = x-8$ $\qquad x = u+8$

$du = dx$

$= \int u^9 + 8u^8\,du$

$= \frac{u^{10}}{10} + \frac{8u^9}{9} + C$

$= \frac{1}{10}(x-8)^{10} + \frac{8}{9}(x-8)^9 + C$

F) $\int \tan(4x)\, dx = \int \frac{\sin(4x)}{\cos(4x)}\, dx = \int \frac{\sin(4x)}{u} \cdot \frac{-1}{4\sin(4x)}\, du$

$u = \cos(4x)$

$du = -4\sin(4x)\, dx$

$-\frac{1}{4\sin(4x)}\, du = dx$

$= \int -\frac{1}{4} \cdot \frac{1}{u}\, du$

$= -\frac{1}{4} \ln|u| + C$

$= -\frac{1}{4} \ln|\cos(4x)| + C$

$= \frac{1}{4} \ln|(\cos(4x))^{-1}| + C$

$= \frac{1}{4} \ln\left|\frac{1}{\cos(4x)}\right| + C$

$= \frac{1}{4} \ln|\sec(4x)| + C$

$$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$$

G) $\int \frac{1+4x}{1+x^2}\,dx = \int \frac{4x}{1+x^2}\,dx + \int \frac{1}{1+x^2}\,dx$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2x}\,du = dx$

$$= \int \frac{4x}{u} \cdot \frac{1}{2x}\,du + \arctan(x) + C$$

$$= \int \frac{2}{u}\,du + \arctan(x) + C$$

$$= 2\ln|u| + \arctan(x) + C$$

$$= 2\ln|1+x^2| + \arctan(x) + C$$

The substitution Rule for Definite Integrals If $g'(x)$ is differentiable on $[a, b]$ and f is continuous on the range of g, then continuous on I, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

$u = g(x)$

$du = g'(x)\,dx$

$\underline{x = a}$ $u = g(a)$

$\underline{x = b}$ $u = g(b)$

Example: Compute

$$\int_0^5 x\cos(0.1x^2 - 1)dx = \int_{-1}^{1.5} x\cos(u)\cdot\frac{5}{x}\,du$$

$$= \int_{-1}^{1.5} 5\cos(u)\,du$$

$u = .1x^2 - 1$

$du = .2x\,dx$

$\frac{1}{.2x}\,du = dx$

$\frac{1}{\frac{1}{5}x}\,du = dx$

$\frac{5}{x}\,du = dx$

$\underline{x = 0}$

$u = -1$

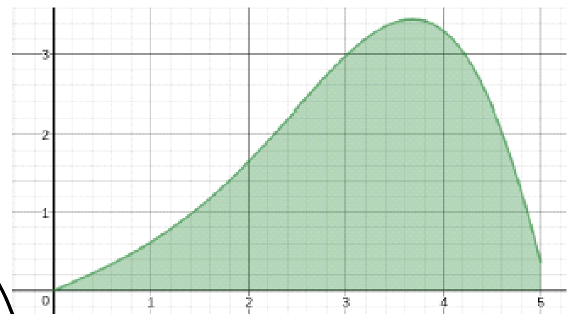
$\underline{x = 5}$

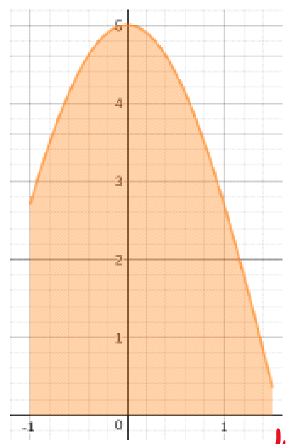
$u = .1(5)^2 - 1$

$u = .1(25) - 1$

$u = 2.5 - 1$

$u = 1.5$





1.5

$$5\sin(u)\Big|_{-1}^{1.5}$$

$$= 5\sin(1.5) - 5\sin(-1)$$

Example: Compute

$$\int_1^2 12x(2x^2+1)^3\,dx = \quad = \int_3^9 12x\,u^3 \cdot \frac{1}{4x}\,du = \int_3^9 3u^3\,du$$

$u = 2x^2+1$

$du = 4x\,dx$

$\frac{1}{4x}\,du = dx$

$\underline{x=1}$

$u = 2(1)^2+1 = 3$

$\underline{x=2}$

$u = 2(2)^2+1 = 9$

$$= \frac{3u^4}{4}\Big|_3^9$$

$$= \frac{3}{4}(9)^4 - \frac{3}{4}(3)^4$$

Example: Compute

$$\int_1^2 12x(2x^2+1)^3\,dx = \quad = \int_{x=1}^{x=2} 12x\,u^3 \cdot \frac{1}{4x}\,du = \int_{x=1}^{x=2} 3u^3\,du$$

$u = 2x^2+1$

$du = 4x\,dx$

$\frac{1}{4x}\,du = dx$

$$= \frac{3u^4}{4}\Big|_{x=1}^{x=2} = \frac{3}{4}(2x^2+1)^4\Big|_1^2$$

$$= \frac{3}{4}(2(2)^2+1)^4 - \frac{3}{4}(2(1)^2+1)^4$$

$$= \frac{3}{4}(9)^4 - \frac{3}{4}(3)^4$$

Example: Compute

$$\int_0^4 xe^{-x^2}\,dx = \int_0^{-16} x e^u \frac{-1}{2x}\,du = \int_0^{-16} -\frac{1}{2} e^u\,du = -\frac{1}{2} e^u \Big|_0^{-16}$$

$u = -x^2$

$du = -2x\,dx$

$\frac{-1}{2x}\,du = dx$

$\underline{x=0}$ $\underline{x=4}$

$u = 0$ $u = -4^2 = -16$

$$= \frac{-1}{2} e^{-16} - \frac{-1}{2} e^0$$

$$= \underbrace{\frac{-1}{2} e^{-16} + \frac{1}{2}}$$

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