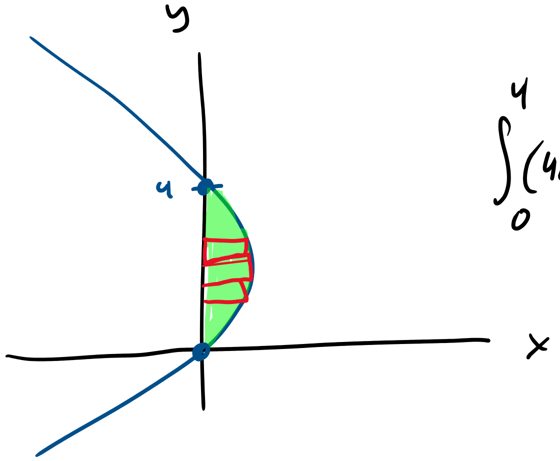
Sections 5.4: Indefinite Integrals and the Net Change Theorem

Example: Sketch the area enclosed by $x = 4y - y^2$ and $x = 0$ and then find the area.

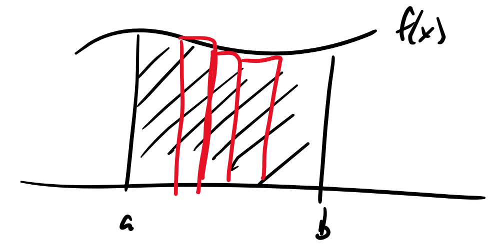
$$x = 4y - y^2$$
$$= y(4 - y)$$

if $x = 0$

$y = 0 \quad y = 4$



$$\int_0^4 (4y - y^2)\,dy$$



$$\int_a^b f(x)\,dx = \text{Area}$$

$$\int_0^4 (4y - y^2)\,dy = \frac{4y^2}{2} - \frac{y^3}{3} \Big|_0^4$$

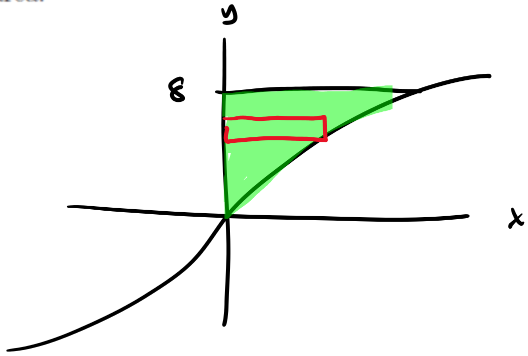
$$= 2(4)^2 - \frac{4^3}{3} - (0) = 32 - \frac{64}{3} = \frac{96}{3} - \frac{64}{3}$$

$$= \frac{32}{3}$$

$y = \sqrt[3]{x}$

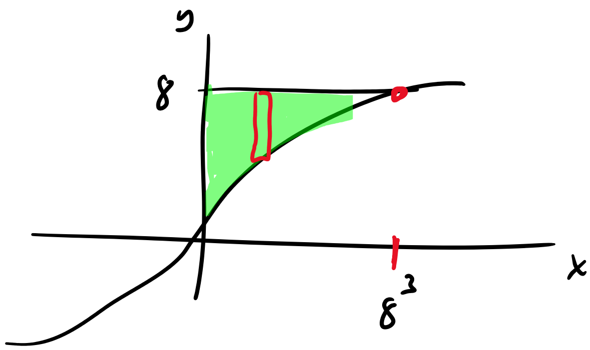
$y^3 = x$

Example: Sketch the area enclosed by $y = \sqrt[3]{x}$, $x = 0$, and $y = 8$ and then find the area.



$$\text{Area} = \int_0^8 y^3 \, dy = \frac{1}{4} y^4 \Big|_0^8 = \frac{1}{4}(8)^4$$

$y = x^{1/3} \rightarrow 8 = x^{1/3}$



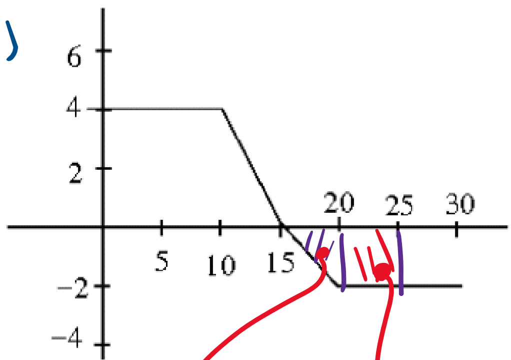
$$\text{Area} = \int_0^{8^3} (8 - x^{1/3}) \, dx$$

Net Change Theorem The integral of a rate of change is the net change:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

Example: Use the graph of $f'(x)$ to answer these questions.

$f'(x)$



$\frac{1}{2}(5)(2)$
$= 5$

$5(2) = 10$

A) Which is larger? $f(15)$ or $f(25)$

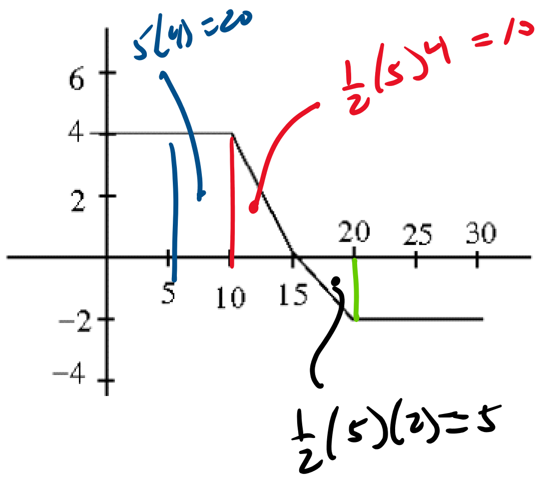
$$\int_{15}^{25} f'(x)\,dx = f(25) - f(15)$$

$$-5 - 10 = f(25) - f(15)$$

$$-15 = f(25) - f(15)$$

$$f(15) = 15 + f(25)$$

Example: Use the graph of $f'(x)$ to answer these questions.



B) Which is larger? $f(10)$ or $f(20)$

$$\int_{10}^{20} f'(x)\,dx = f(20) - f(10)$$

$$10 - 5 = f(20) - f(10)$$

$$5 = f(20) - f(10)$$

$$f(10) + 5 = f(20)$$

C) If $f(5) = 30$, find $f(20)$.

$$\int_{5}^{20} f'(x)\,dx = f(20) - f(5)$$

$$20 + 10 - 5 = f(20) - f(5)$$

$$25 = f(20) - 30$$

$$55 = f(20)$$

Example: A particle is moving in straight line motion that is expressed by the formula: $v(t) = t^2 - t - 6$ (measured in meters per second).

A) Find the displacement from $t = 1$ to $t = 4$.

$$\int_1^4 v(t)\,dt = s(t)\Big|_1^4 = s(4) - s(1)$$

$$\int_1^4 (t^2 - t - 6)\,dt = \left(\frac{t^3}{3} - \frac{t^2}{2} - 6t\right)\Big|_1^4 = \frac{64}{3} - \frac{16}{2} - 24 - \left(\frac{1}{3} - \frac{1}{2} - 6\right)$$

$$= \frac{64}{3} - 32 - \frac{1}{3} + \frac{1}{2} + 6$$

$$= \frac{63}{3} - 26 + \frac{1}{2} = 21 - 26 + \frac{1}{2} = -5 + \frac{1}{2} = \underline{-4.5 \text{ meters}}$$

ended up. 4.5 meters to the left of $s(1)$

$v(t) = t^2 - t - 6$

$0 = (t-3)(t+2)$

$t = 3 \quad t = -2$

B) Find the total distance traveled from $t = 1$ to $t = 4$.

$$\left|\int_1^3 v(t)\,dt\right| + \int_3^4 v(t)\,dt$$

$$-\int_1^3 (t^2 - t - 6)\,dt + \int_3^4 (t^2 - t - 6)\,dt = \cdots = \frac{61}{6} \text{ meters}$$