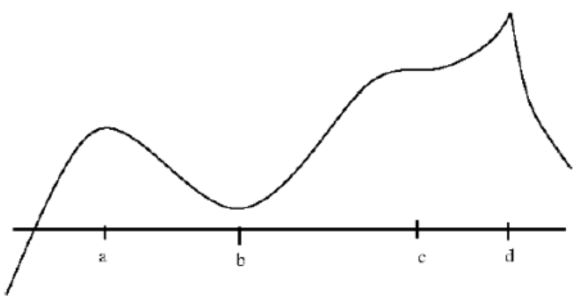
Section 4.1-4.3 Part 1: What Does f' and f'' say about f?

Definition: A function is said to be **increasing** on an interval if for a and b in the interval with $a < b$, then $f(a) < f(b)$. A function is said to be **decreasing** on an interval if for a and b in the interval with $a < b$, then $f(a) > f(b)$.



Inc: $(-\infty, a)$

(b, c)

(c, d) → (b, d)

Dec: (a, b)

(d, ∞)

critical values:

$f'(x) = 0$ at $x = a$, $x = b$, $x = c$

$f'(x)$ DNE at $x = d$

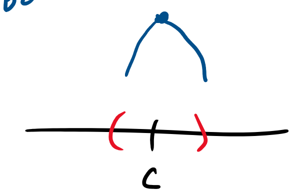
local min @ $x = b$

local max @ $x = a$, $x = d$

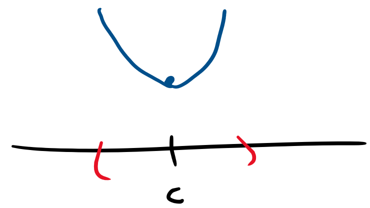
Definition: A **critical number (critical value)** is a number, c, in the domain of f such that

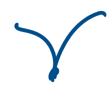
$f'(c) = 0$ or $f'(c)$ DNE

local max

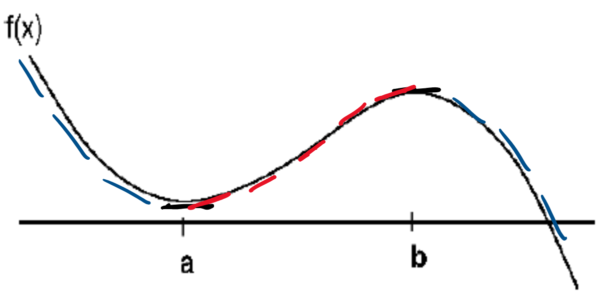


Definition: A function has a **local maximum (relative maximum)** at c if $f(c) \geq f(x)$ on an open interval that contains c, i.e. when x is near c. The value of the local maximum is $f(c)$.
Similarly, a function has a **local minimum (relative minimum)** at c if $f(c) \leq f(x)$ on an open interval that contains c. The value of the local minimum is $f(c)$.





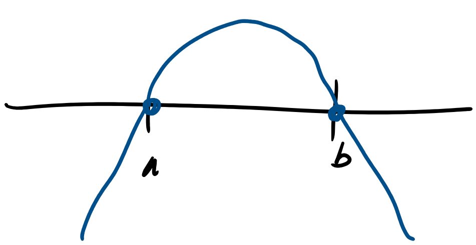
Discuss the properties of the the derivate $f'(x)$ and how it relates to the properties of $f(x)$.



$f(x)$
Inc (a, b) $\quad f'(x) > 0$

$f(x)$
dec $(-\infty, a)$ $\quad f'(x) < 0$
(b, ∞)

Sketch graph of $f'(x)$



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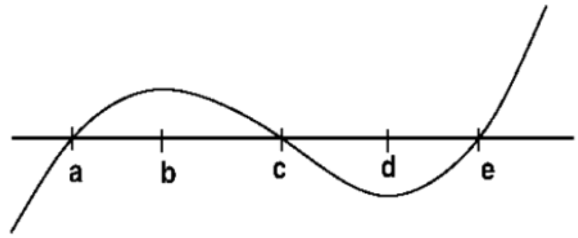
Example: Here is the graph of $f'(x)$.

$f'(x)$



A) Where is $f(x)$ increasing? → $f'(x) > 0$

graph of $f'(x)$ is above x-axis

(a, c)

(e, ∞)

B) Where is $f(x)$ decreasing? → $f'(x) < 0$ → graph of $f'(x)$ is below x-axis.

$(-\infty, a)$ (c, e)

C) Where does $f(x)$ have a local minimum?

$f(x)$

Dec/Inc.

$x = a$

$x = e$

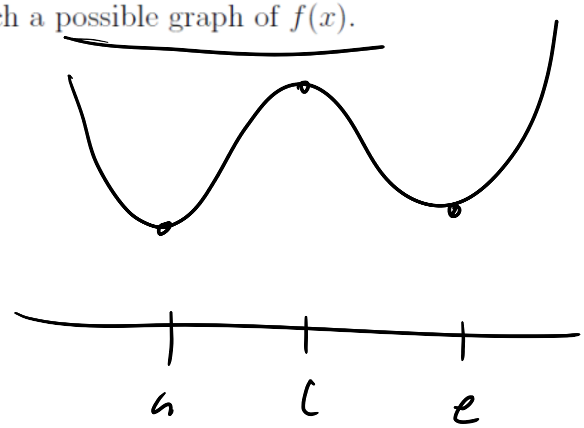
D) Where does $f(x)$ have a local maximum?

$f(x)$

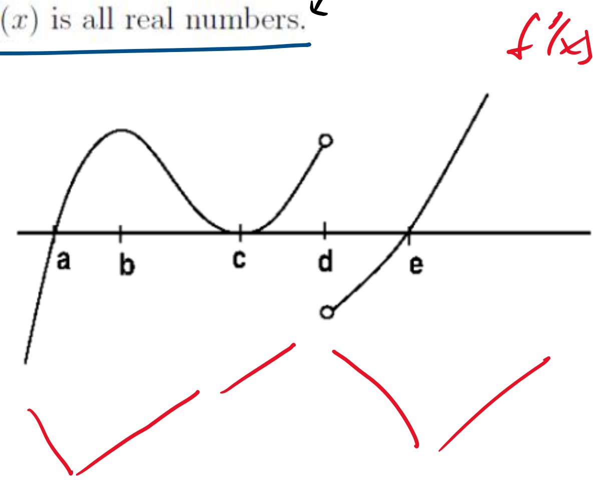
Inc/Dec.

$x = c$

E) Sketch a possible graph of $f(x)$.



Example: Here is the graph of $f'(x)$. The domain of $f(x)$ is all real numbers. ← $f(x)$ is continuous.



A) Where is $f(x)$ increasing? → $f'(x) > 0$

(a, c)

(c, d) (e, ∞)

B) Where is $f(x)$ decreasing? → $f'(x) < 0$

$(-\infty, a)$ (d, e)

C) Where does $f(x)$ have a local minimum? $f(x)$

$x = a$ $x = e$

Dec/Inc

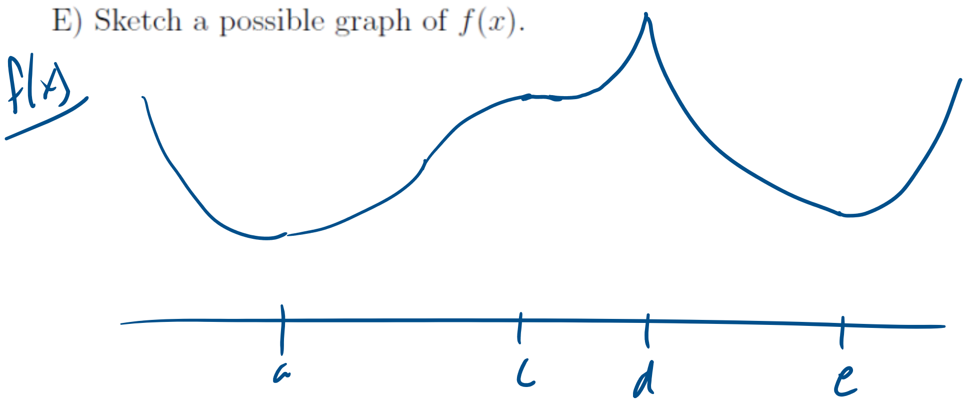
D) Where does $f(x)$ have a local maximum? $f(x)$

local max at

$x = d$

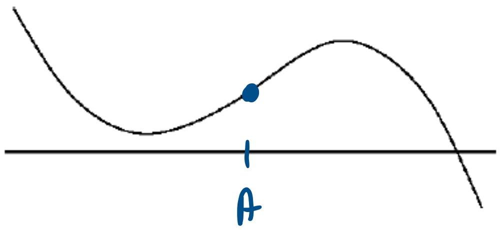
Inc/Dec.

E) Sketch a possible graph of $f(x)$.

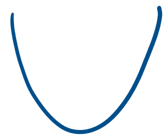


Example: Examine the concavity of the function $f(x)$.

Concave up Concave down

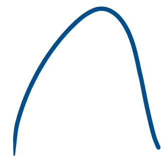


A



Concave up

c.u.



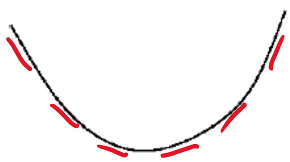
Concave down

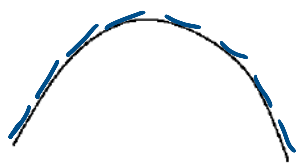
c.d.

Definition: An **inflection point** is a point on the graph of $f(x)$ where $f(x)$ changes concavity.

Discuss the properties of the the derivate $f''(x)$ and how it relates to concavity of $f(x)$.

$+ \quad +$ $\quad - \quad -$



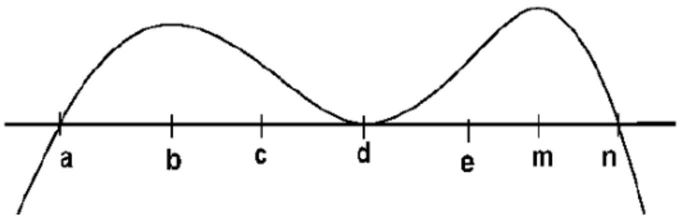


Concave down

Concave up

$$\boxed{\begin{array}{l} f(x) \text{ c.u.} \iff f'(x) \text{ is inc.} \iff f''(x) > 0 \\ f(x) \text{ c.d} \iff f'(x) \text{ is dec} \iff f''(x) < 0 \end{array}}$$

Example: Here is the graph of $f''(x)$. $f''(x)$



$f(x)$ c.u. $\Leftrightarrow$ $f''(x) > 0$

$\Updownarrow$

graph of $f''(x)$ is above the x-axis.

$f(x)$ c.d $\Leftrightarrow$ $f''(x) < 0$

$\hookrightarrow$ graph of $f''(x)$ is below the x-axis

A) Where is $f(x)$ concave up?

(a, d) (d, n)

B) Where is $f(x)$ concave down?

$(-\infty, a)$ (n, ∞)

C) Find all x−values of the inflection points. for $f(x)$.

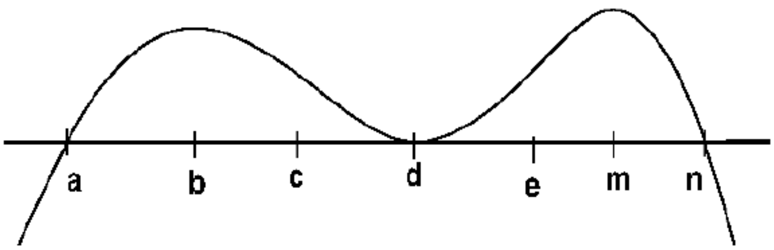
$x = a$ + $x = n$

Example: Here is the graph of $f'(x)$.

$f'(x)$

$f(x)$ c.u. $\Leftrightarrow$ $f'(x)$ Inc.

$f(x)$ c.d $\Leftrightarrow$ $f'(x)$ dec.



A) Where is $f(x)$ concave up?

$(-\infty, b)$

(d, m)

B) Where is $f(x)$ concave down?

(b, d) (m, ∞)

C) Find all $x-$values of the inflection points. for $f(x)$

$x = b, d, m$

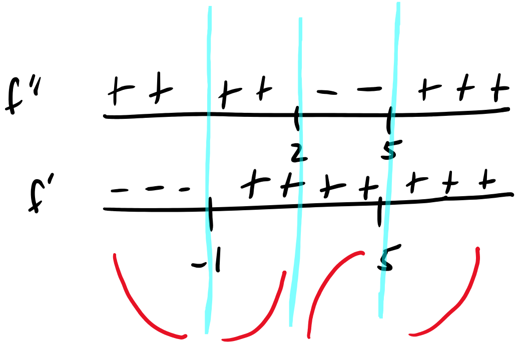
Example: Sketch the graph of a function that meets these conditions.

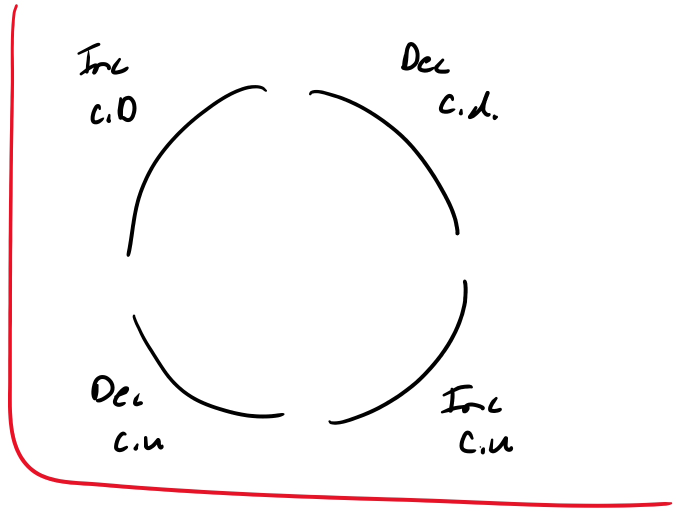
Continuous and differentiable for all real numbers.
$f'(-1) = 0$ and $f'(5) = 0$

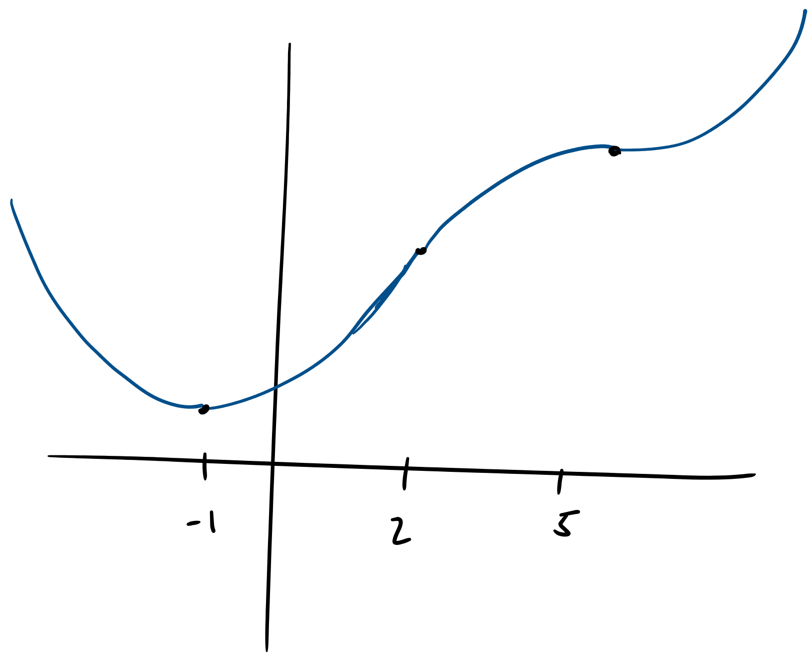
Critical values
$x = -1$
$x = 5$
horizontal slopes

$f'(x) > 0$ on $(-1, 5)$, $(5, \infty)$
$f'(x) < 0$ on $(-\infty, -1)$

$f''(x) > 0$ on $(-\infty, 2)$, $(5, \infty)$
$f''(x) < 0$ on $(2, 5)$







point.

Example: Sketch the graph of a function that meets these conditions.

$f'(1) = 0$, $\boxed{f(0) = 1}$ $\underbrace{\lim_{x \to \infty} f(x) = 3}$ H.A.

$f'(x) > 0$ on $(0, 1)$
$f'(x) < 0$ on $(-\infty, 0)$, $(1, \infty)$

$f''(x) < 0$ on $(0, 2)$
$f''(x) > 0$ on $(-\infty, 0)$, $(2, \infty)$