$\tan x = \frac{\sin(x)}{\cos(x)}$

Section 3.3: Derivatives of Trigonometric Functions

Derivative Rules

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

Example: Find the derivatives of these functions.

A) $y = 5\tan(x) + 3\sec(x)$

$y' = 5\sec^2(x) + 3\sec(x)\tan(x)$

$\csc^2(x) \neq \csc(x)^2 \neq \csc x^2$ ✗

$\csc^2(x) = (\csc(x))^2$ ✓

B) $y = x^3 \cot(x)$

$y' = 3x^2 \cot(x) + x^3(-\csc^2(x))$

$= 3x^2 \cot(x) - x^3 \csc^2(x)$

C) $y = \dfrac{\sin(x)}{1+\csc(x)}$

$$y' = \frac{(1+\csc x)\cos(x) - \sin(x)(-\csc x \cot x)}{(1+\csc x)^2}$$

Example: Compute $\frac{d^{99}}{dx^{99}}\sin(x)$

The 99th deriv.

$$4\overline{)99} \quad 24 \quad 8 \quad 19 \quad 16 \quad 3$$

Remainder of 3

$y = \sin(x)$ $\quad y^{(4)}$ $\quad y^{(8)}$

$y' = \cos(x)$ $\quad y^{(5)}$

$y'' = -\sin(x)$ $\quad y^{(6)}$

$\rightarrow$ $y''' = -\cos(x)$ $\quad y^{(7)}$

$y^{(4)} = -(-\sin(x)) = \sin(x)$

$$\frac{d^{99}}{dx^{99}}\sin(x) = -\cos(x)$$

Example: Find where the tangent line is horizontal.

$y = x + 2\cos(x)$

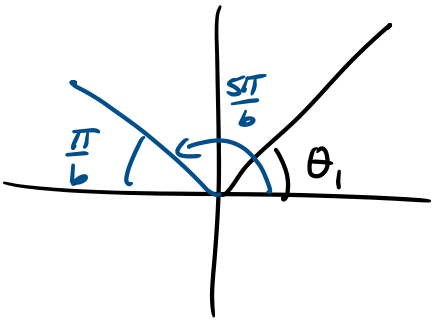
↳ $m_{tan} = 0$ ie where is $y' = 0$?

$y' = 1 - 2\sin(x)$

$0 = 1 - 2\sin(x)$

$2\sin(x) = 1$

$\sin(x) = \frac{1}{2}$



$\theta_1 = \frac{\pi}{6}$ $\theta_2 = \frac{5\pi}{6}$

$\theta_1 = \frac{\pi}{6} + 2n\pi$ $\theta_2 = \frac{5\pi}{6} + 2n\pi$

where n is an Integer.