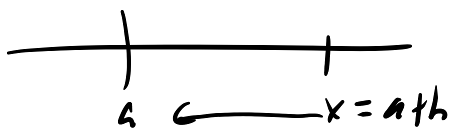
Section 2.8: Derivative

Definition: The **derivative of a function** f at a number a, denoted $f'(a)$, is

$$m_{tan} = f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



$$\frac{f(a+h) - f(a)}{a+h-a} = \frac{f(a+h) - f(a)}{h}$$

$$f(x) = \frac{2}{x+5}$$

$$f(2) = \frac{2}{2+5} = \frac{2}{7}$$

$$f(☺) = \frac{2}{☺+5}$$

$$f(\square) = \frac{2}{\square+5}$$

Example: Find the derivative of $f(x) = \dfrac{2}{x+5}$ at $a = 0$, $a = 2$, $a = 3$, $a = -5$.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2}{2+h+5} - \frac{2}{2+5}}{h} = \lim_{h\to 0} \frac{\frac{2 \cdot 7}{(7+h)7} - \frac{2(7+h)}{7(7+h)}}{\frac{h}{1}}$$

$$= \lim_{h\to 0} \frac{\frac{14}{7(7+h)} - \frac{14+2h}{7(7+h)}}{\frac{h}{1}} = \lim_{h\to 0} \frac{\frac{14-14-2h}{7(7+h)}}{\frac{h}{1}}$$

$$= \lim_{h\to 0} \frac{-2h}{7(7+h)} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{-2}{7(7+h)} = \frac{-2}{7(7+0)} = \frac{-2}{49}$$

$$f(x) = \frac{2}{x+5}$$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{2}{x+h+5} - \frac{2}{x+5}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2(x+5)}{(x+h+5)(x+5)} - \frac{2(x+h+5)}{(x+h+5)(x+5)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2x+10-2x-2h-10}{(x+h+5)(x+5)}}{\frac{h}{1}} = \lim_{h\to 0} \frac{-2h}{(x+h+5)(x+5)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-2}{(x+h+5)(x+5)} = \frac{-2}{(x+5)^2} = f'(x)$$

$h \to 0 \quad (x+h+5)(x+5) \qquad (x+5)^2$

$$f'(2) = \frac{-2}{(2+5)^2} = \frac{-2}{49}$$

$$f'(0) = \frac{-2}{5^2} = \frac{-2}{25}$$

$$f'(3) = \frac{-2}{8^2} = \frac{-2}{64}$$

$f'(-5) =$ DNE

Definition of the Derivative: The derivative of a function $f(x)$, denoted $f'(x)$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Other common notations for the derivative are f', $\frac{dy}{dx}$, and $\frac{d}{dx}f(x)$

Note: Once you have the function $f'(x)$, also called the **first derivative**, you can redo the derivative process with that function and compute the **second derivative** (notation: $f''(x)$, y'', $\frac{d^2y}{dx^2}$...).

Example: For the function $f(x) = \frac{2}{x+5}$, find the equation of the tangent line at $x = 3$.

$f(3) = \frac{2}{8}$

$f'(x) = \frac{-2}{(x+5)^2}$

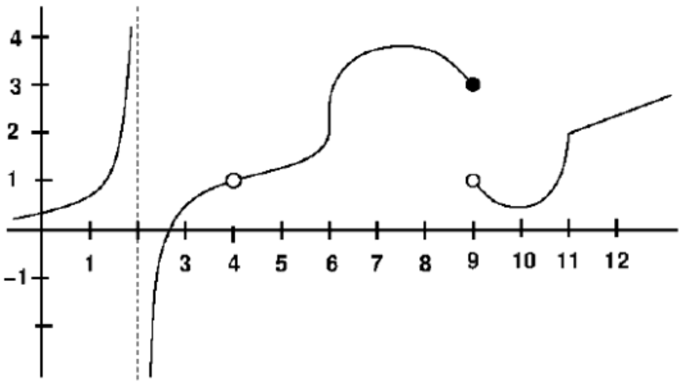
$m_{tan} = f'(3) = \frac{-2}{8^2} = \frac{-2}{64}$

Tangent line

$y - f(3) = f'(3)(x-3)$

$y - \frac{2}{8} = \frac{-2}{64}(x-3)$

Example: Here is the graph of $f(x)$. Where does the derivative not exist?



$f'(x)$ DNE

$x=2$, $x=4$ not continuous
$x=9$

$x=11$ sharp point.

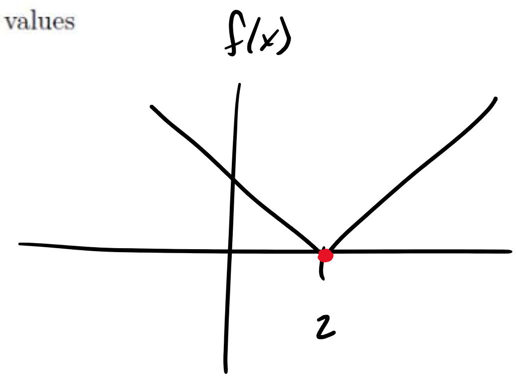
$x=6$ vertical tangent line.

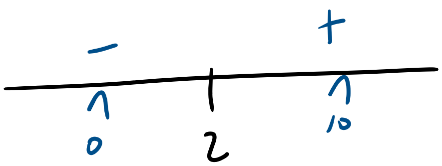
Definition: $f(x)$ is said to be **differentiable** at $x = a$ provided that $f'(a)$ exists. $f(x)$ is differentiable on an open interval (a, b) provided it is differentiable at every number in the interval.

Theorem: If f is differentiable at a, then f is continuous at a.

Example: Sketch the graph of $f(x)$ and use this graph to find $f'(x)$. Give the values where $f(x)$ is not continuous and where it is not differentiable.

$$f(x) = |2x-4| = \begin{cases} (2x-4), & x \geq 2 \\ -(2x-4), & x < 2 \end{cases}$$



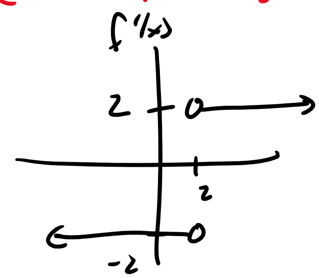


f(x) is continuous. :)

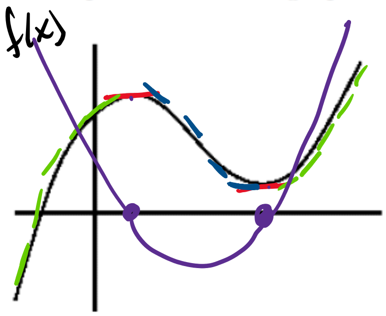
f'(x) DNE at x=2 (sharp point.)

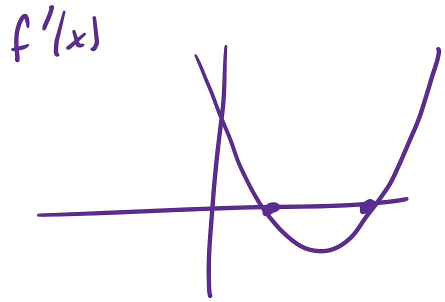
f(x) is not differentiable at x=2

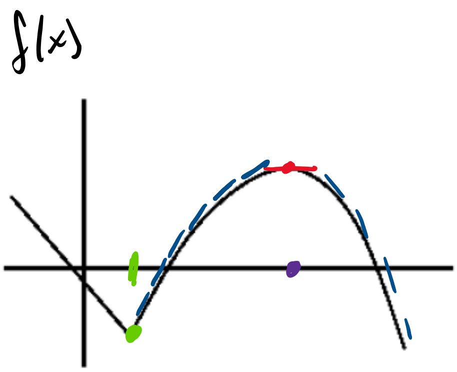
$$f'(x) = \begin{cases} 2, & x > 2 \\ -2, & x < 2 \end{cases}$$

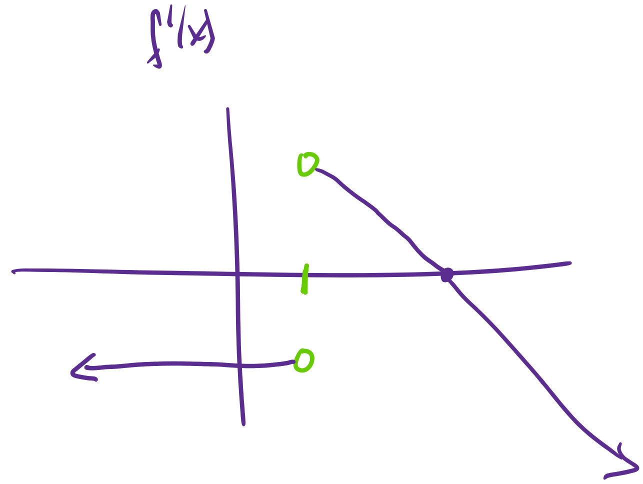


Example: Sketch the graph of the derivative for these graphs.









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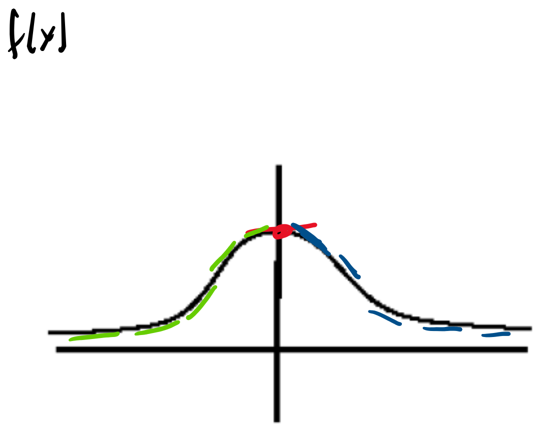

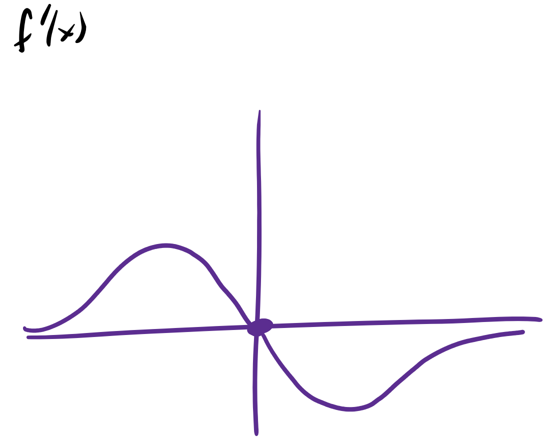

$g(\square) = 3\square^2 + 2\square + 7$

Example: Use the definition of the derivative to find $g'(x)$ for $g(x) = 3x^2 + 2x + 7$

$$\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 + 2(x+h) + 7 - (3x^2 + 2x + 7)}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + 2x + 2h + 7 - 3x^2 - 2x - 7}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{2x} + 2h + \cancel{7} - \cancel{3x^2} - \cancel{2x} - \cancel{7}}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2 + 2h}{h} = \lim_{h \to 0} \frac{\cancel{h}(6x + 3h + 2)}{\cancel{h}}$$

$6x + 3(0) + 2$

$$= \lim_{h \to 0} 6x + 3h + 2 = 6x + 2 = g'(x)$$

Example: Use the definition of the derivative to find $g'(x)$ for $g(x) = \sqrt{3x+5}$.

$$\lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = \lim_{h\to 0} \frac{\sqrt{3(x+h)+5} - \sqrt{3x+5}}{h}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{3x+3h+5} - \sqrt{3x+5}\right)}{h} \cdot \frac{\left(\sqrt{3x+3h+5} + \sqrt{3x+5}\right)}{\left(\sqrt{3x+3h+5} + \sqrt{3x+5}\right)}$$

$$= \lim_{h\to 0} \frac{3x+3h+5-(3x+5)}{h\left(\sqrt{3x+3h+5} + \sqrt{3x+5}\right)} = \lim_{h\to 0} \frac{\cancel{3x}+3h\cancel{+5}-\cancel{3x}\cancel{-5}}{h\left(\sqrt{3x+3h+5} + \sqrt{3x+5}\right)}$$

$$= \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}\left(\sqrt{3x+3h+5} + \sqrt{3x+5}\right)} = \lim_{h\to 0} \frac{3}{\sqrt{3x+3h+5} + \sqrt{3x+5}}$$

$$= \frac{3}{\sqrt{3x+5} + \sqrt{3x+5}} = \frac{3}{2\sqrt{3x+5}} = g'(x)$$