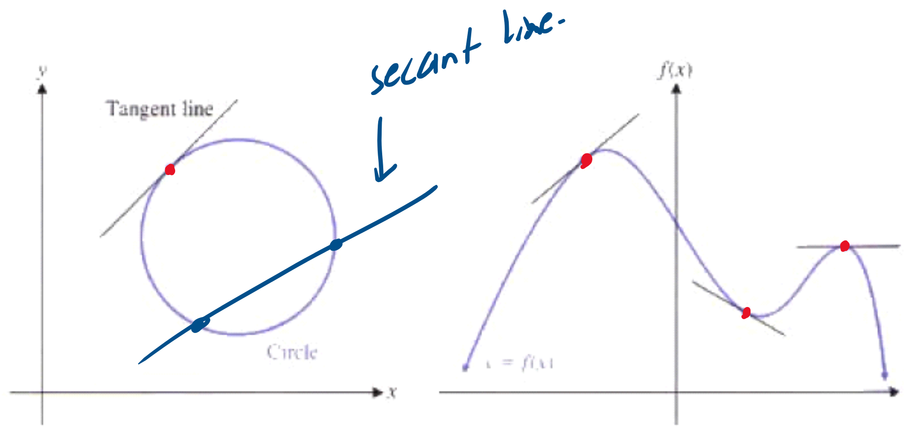
## Section 2.7: Tangents, Velocities, and Other Rates of Change

$f'(a) = m_{tan}\Big|_{x=a}$

**Definition:** The **instantaneous rate of change** of a function $f(x)$ at $x = a$ is the slope of the **tangent line** at $x = a$ and is denoted $f'(a)$.




(A) Tangent line for a circle

(B) Tangent lines for the graph of a function

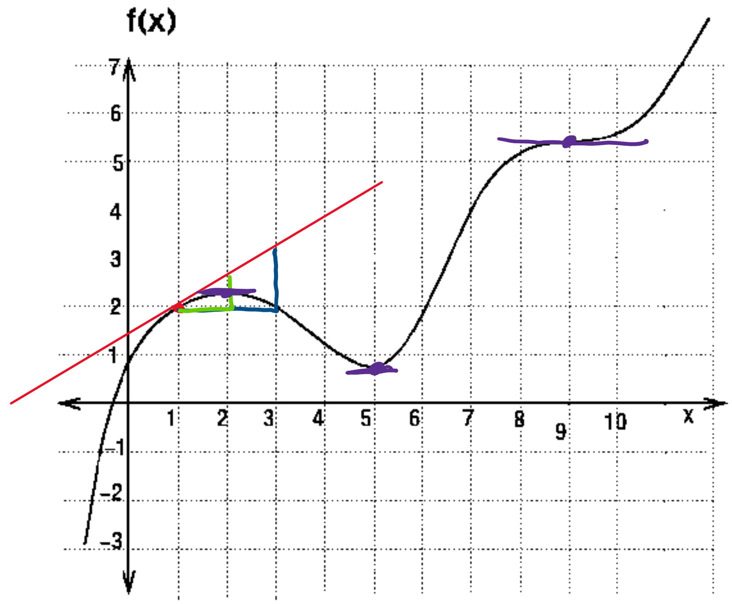
Example: Use this graph to answer these questions.

$f(1) = 2$

A) Estimate the instantaneous rate of change at $x = 1$.

$m_{tan} \approx \frac{1.25}{2} = .625$ ← use this one.

$m_{tan} \approx \frac{.75}{1}$




B) Find the equation of the tangent line at $x = 1$.

$y - y_1 = m(x - x_1)$   $f(1) = 2$

$y - 2 = .625(x - 1)$

C) At what values of $x$ does $f(x)$ have an instantaneous rate of change of 0?

$x = 5$, $x = 2$, $x = 9$

↳ $m_{tan}$ is horizontal

slope of the secant line. (slope of line through the 2 points)

Example: Find the average rate of change of $f(x) = 2x^2 - x$ from

A) $x = 1$ to $x = 6$

$$\frac{f(6)-f(1)}{6-1} = \frac{(2(6)^2-6)-(2(1)^2-1)}{5} = \frac{(72-6)-1}{5}$$

$$= \frac{66-1}{5} = \frac{65}{5} = 13$$

B) $x = 1$ to $x = 5$

$50-5=45$

$$\frac{f(5)-f(1)}{5-1} = \frac{45-1}{4} = \frac{44}{4} = 11$$

C) $x = 1$ to $x = 3$

$$\frac{f(3)-f(1)}{3-1} = \frac{(18-3)-1}{2} = \frac{15-1}{2} = \frac{14}{2} = 7$$

**Definition:** The slope of the tangent line(instantaneous rate of change) at $x = a$ is

$$m_{tan} = \lim_{x \to a} \underbrace{\frac{f(x) - f(a)}{x - a}}_{\text{avg. Rate of Change}}$$

Example: Find the slope of the tangent line for $f(x) = 2x^2 - x$ at $x = 1$. $(a=1)$ Also give the equation of the tangent line at $x = 1$.

$f(1) = 2(1)^2 - 1$
$= 2 - 1 = 1$

$$m_{tan} = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{2x^2 - x - 1}{x - 1}$$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(2x+1)}{\cancel{(x-1)}} = \lim_{x \to 1} 2x + 1 = 2(1) + 1$$

$$= 3$$

eq. of the tangent line

$$y - f(1) = m_{tan}(x - 1)$$

$$y - 1 = 3(x - 1)$$

$y - 1 = 3x - 3$

$y = 3x - 2$

$(x=a)$

Example: Find the instantaneous rate of change at $x = 9$ for $f(x) = \sqrt{x}$.

$$m_{tan} = f'(9) = \lim_{x \to 9} \frac{f(x) - f(9)}{x - 9} = \lim_{x \to 9} \frac{(\sqrt{x} - 3)}{(x - 9)} \cdot \frac{(\sqrt{x} + 3)}{(\sqrt{x} + 3)}$$

$$= \lim_{x \to 9} \frac{x - 9}{(x - 9)(\sqrt{x} + 3)} = \lim_{x \to 9} \frac{1}{\sqrt{x} + 3}$$

$$= \frac{1}{3 + 3} = \frac{1}{6}$$

$$x - 9 = (\sqrt{x})^2 - 3^2 = (\sqrt{x} - 3)(\sqrt{x} + 3)$$