Pg 1

Suppose that a curve, C, is described by the parametric equations $x = x(t)$ and $y = y(t)$ or the vector function $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ where both $x(t)$ and $y(t)$ are differentiable. Then the slope of the tangent line is given by

$$\text{slope} = \frac{y'(t)}{x'(t)}.$$

$$r(t) = \langle x(t), y(t) \rangle$$

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$$\text{slope} = \frac{y'(t)}{x'(t)}$$

Suppose.

$$r'(1) = \langle 3, 4 \rangle$$

$$m = \frac{4}{3}$$

$$\frac{dy}{dx} = \frac{dy/dt}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)}$$

Example: Find $\frac{dy}{dx}$ and $\left.\frac{dy}{dx}\right|_{t=3}$ and $\left.\frac{dy}{dx}\right|_{(5,-1)}$ x y

$x(t) = t^3 - 3t^2 + 5$
$y(t) = 2t - 7$

$y = 2t - 7$

$\frac{y+7}{2} = t$

$x = \left(\frac{y+7}{2}\right)^3 - 3\left(\frac{y+7}{2}\right)^2 + 5$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)} = \frac{2}{3t^2 - 6t}$$

$$\left.\frac{dy}{dx}\right|_{t=3} = \frac{2}{3(3)^2 - 6(3)} = \frac{2}{27-18} = \frac{2}{9} = \left.\frac{dy}{dx}\right|_{\underbrace{(5,-1)}_{t=3}}$$

Solve for t

point $(\underline{5}, -1)$

$x=5$: $t^3 - 3t^2 + 5 = 5$ ✓

$y=-1$: $2t - 7 = -1$

$2t = 6$

$t = 3$ ✓ !!

$x = (3)^3 - 3(3)^2 + 5$

$x = 27 - 27 + 5 = 5$ ✓

$t^3 - 3t^2 = 0$

$t^2(t-3) = 0$

$\underbrace{t=0}$ $\underbrace{t=3}$

Example: Find the equation of the tangent line at $t = 0$.

$x(t) = e^{t^2+4t}$
$y(t) = 5^{3t+2}$

point $x(0) = e^0 = 1$
$y(0) = 5^2 = 25$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{3 \cdot 5^{3t+2} \cdot \ln(5)}{(2t+4) \cdot e^{t^2+4t}}$$

$$m_{tan} = \left.\frac{dy}{dx}\right|_{t=0} = \frac{3 \cdot 5^2 \ln(5)}{4e^0} = \frac{75\ln(5)}{4}$$

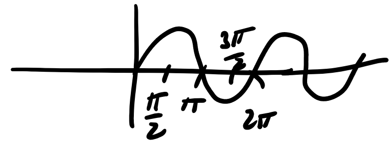
tangent line: $\boxed{y - 25 = \frac{75\ln(5)}{4}(x-1)}$

Example: Compute the derivatives at the point $(0, 0)$.
$x(t) = \sin(2t)$
$y(t) = \cos(t)$

find values of t

$x = 0$

$0 = \sin(2t)$

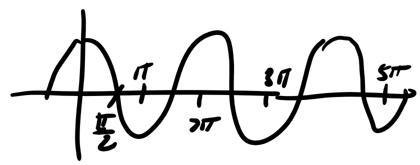


$2t = \ldots, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$

$t = \ldots, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, \ldots$

$y = 0$

$0 = \cos(t)$



$t = \ldots, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

point $(0,0)$ is found for

$t = \ldots, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{-\sin(t)}{2\cos(2t)}$$

$$\left.\frac{dy}{dx}\right|_{t=\pi/2} = \frac{-(1)}{2(-1)} = \frac{1}{2}$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{3\pi}{2}} = \frac{-(-1)}{2(-1)} = -\frac{1}{2}$$

$t = \frac{7\pi}{2}$

$$\left.\frac{dy}{dx}\right|_{t=\frac{5\pi}{2}} = \frac{-(1)}{2(-1)} = \frac{1}{2}$$

Horizontal tangent lines $\to m = 0$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$$

$y'(t) = 0$ and $x'(t) \neq 0$

Vertical tangent lines $m =$ undefined.

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$$

$x'(t) = 0$ and $y'(t) \neq 0$

Example: Find the points on the curve where the tangent lines are horizontal and where they are vertical.

$x = t^2 + t$
$y = t^2 - t$

$y'(t) = 2t - 1$

$0 = 2t - 1$

$t = \frac{1}{2}$

$x'(t) = 2t + 1$

$0 = 2t + 1$

$t = \frac{-1}{2}$

Horizontal tangent line at $t = \frac{1}{2}$

Vertical tangent line at $t = \frac{-1}{2}$

Points

$x(\frac{1}{2}) = (\frac{1}{2})^2 + \frac{1}{2} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

$y(\frac{1}{2}) = (\frac{1}{2})^2 - \frac{1}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

$(\frac{3}{4}, \frac{-1}{4})$

$x(\frac{-1}{2}) = \frac{1}{4} + \frac{-1}{2} = -\frac{1}{4}$

$y(\frac{-1}{2}) = \frac{1}{4} - \frac{-1}{2} = \frac{3}{4}$

$(\frac{-1}{4}, \frac{3}{4})$

Example: Find the values of t where the tangent lines are horizontal and where they are vertical.

$x = t + 3$
$y = t^3 - 3t^2$

$y' = 3t^2 - 6t$

$0 = 3t^2 - 6t$

$0 = 3t(t-2)$

$t = 0 \quad t = 2$

Horizontal tanget lines

$t = 0, t = 2$

$x' = 1$

set $x' = 0$ no solution

so no vetical tangent lines.