Word Problems for Section 2.3

Solve the following word problems. Be sure to tell what restrictions can be placed on the parameters.

1. The accompanying figure shows the flow of downtown traffic in a certain city during rush hours on a typical weekday. The arrows indicate the direction of traffic flow on each one–way road, and the average number of vehicles entering and leaving each intersection per hour appears beside each road. Fifth and Sixth Avenues can handle up to 2000 vehicles per hour without causing congestion, whereas the maximum capacity of each of the two streets is 1000 vehicles per hour. The flow of traffic is controlled by traffic lights installed at each of the four intersections.

   (a) Set up the system of equations that would model this problem.

   (b) Solve the system of equations and write the answer in parametric form. Place restrictions on the parameter.

   (c) Find two possible flow patters that would ensure that there is no traffic congestion.
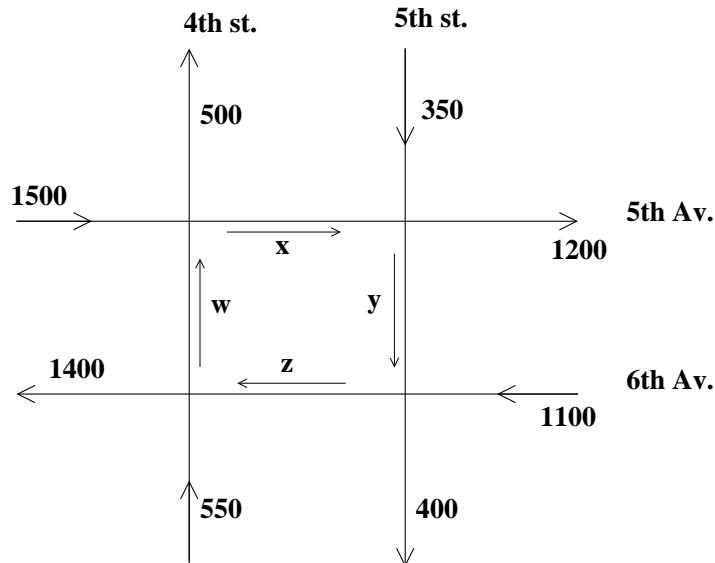
2. A farmer can buy four types of plant food. Each barrel of mix A contains 30 pounds of phosphoric acid, 50 pounds of nitrogen, and 30 pounds of potash; each barrel of mix B contains 30 pounds of phosphoric acid, 75 pounds of nitrogen, and 20 pounds of potash; each barrel of mix C contains 30 pounds of phosphoric acid, 25 pounds of nitrogen, and 20 pounds of potash; and each barrel of mix D contains 60 pounds of phosphoric acid, 25 pounds of nitrogen, and 50 bounds of potash. soil test indicate that a particular field needs 900 pounds of phosphoric acid, 750 pounds of nitrogen, and 700 pounds of potash. How many barrels of each type of food should the farmer buy to supply the necessary nutrients for the field?

3. A chemical manufacturer wants to purchase a fleet of 24 railroad tank cars with a combined carrying capacity of 250,000 gallons. Tank cars with three different carrying capacities are available: 6,000 gallons, 8,000 gallons, and 18,000 gallons. How many of each type of tank car should be purchased?

1. The system of equations was generated by the following thought: what goes into an intersection must come out of an intersection.

Definition of variables:
x = average number of cars on 5th Av.
y = average number of cars on 5th St.
z = average number of cars on 6th Av.
w = average number of cars on 4th St.

System of equations:
$$x - w = 1000$$
$$x - y = 850$$
$$y - z = -700$$
$$z - w = 850$$

Initial matrix

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 1000 \\ 1 & -1 & 0 & 0 & 850 \\ 0 & 1 & -1 & 0 & -700 \\ 0 & 0 & 1 & -1 & 850 \end{bmatrix}$$

$\xrightarrow{\text{rref}}$

final matrix

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 1000 \\ 0 & 1 & 0 & -1 & 150 \\ 0 & 0 & 1 & -1 & 850 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Parametric Solution:
$$x = 1000 + w$$
$$y = 150 + w$$
$$z = 850 + w$$
$$w = any\ number$$

Since this is a word problem, we can place restrictions on the parameter. We know that all of the variables must be greater than or equal to zero.

$$x \geq 0 \qquad\qquad y \geq 0 \qquad\qquad z \geq 0 \qquad\qquad w \geq 0$$
$$100 + w \geq 0 \qquad 150 + w \geq 0 \qquad 850 + w \geq 0$$
$$w \geq -1000 \qquad\quad w \geq -150 \qquad\quad w \geq -850$$

In addition we know that x and z can not be any larger than 2000 and y and w can not be any larger than 1000.

$$x \leq 2000 \qquad\qquad y \leq 1000 \qquad\qquad z \leq 2000 \qquad\qquad w \leq 1000$$
$$100 + w \leq 2000 \qquad 150 + w \leq 1000 \qquad 850 + w \leq 2000$$
$$w \leq 1000 \qquad\qquad w \leq 850 \qquad\qquad w \leq 1150$$

When all of these restrictions are considered, we find that in order to have a valid solution, we need the restriction on the parameter to be $0 \leq w \leq 850$.

2. Definition of variables:
x = number of barrels of mix A
y = number of barrels of mix B
z = number of barrels of mix C
w = number of barrels of mix D

System of equations:
These can be written down by setting up a table.
$$30x + 30y + 30z + 60w = 900$$
$$50z + 75y + 25z + 25w = 750$$
$$30x + 20y + 20z + 50w = 700$$

Initial matrix

$$\begin{bmatrix} 30 & 30 & 30 & 60 & 900 \\ 50 & 75 & 25 & 25 & 750 \\ 30 & 20 & 20 & 50 & 700 \end{bmatrix}$$

$\xrightarrow{\text{rref}}$

final matrix

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & -1 & -5 \\ 0 & 0 & 1 & 2 & 25 \end{bmatrix}$$

Parametric Solution:
$$x = 10 - w$$
$$y = -5 + w$$
$$z = 25 - 2w$$
$$w = any\ number$$

Now place restrictions on the parameter. Since we are not told a maximum number of barrels that can be bought assume there is no limit. We know the number of barrels bought has to be non-negative.

$$x \geq 0 \qquad\qquad y \geq 0 \qquad\qquad z \geq 0 \qquad\qquad w \geq 0$$
$$10 - w \geq 0 \qquad -5 + w \geq 0 \qquad 25 - 2w \geq 0$$
$$10 \geq w \qquad\qquad w \geq 5 \qquad\qquad 25 \geq 2w$$
$$w \leq 10 \qquad\qquad\qquad\qquad\qquad 12.5 \geq w$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad w \leq 12.5$$

The restriction on the parameter is that w must be an integer and $5 \leq w \leq 10$ (i.e. $w = 5$, 6, 7, 8, 9, 10)

3. Definition of variables:

x = number of tank cars purchased with 6,000 gallon capacity
y = number of tank cars purchased with 8,000 gallon capacity
z = number of tank cars purchased with 18,000 gallon capacity

System of equations:

$x + y + z = 24$
$6000x + 8000y + 18000z = 250000$

Initial matrix and final matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 24 \\ 6000 & 8000 & 18000 & 250000 \end{array} \right] \xrightarrow[\rightarrow]{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & \text{-5} & \text{-29} \\ 0 & 1 & 6 & 53 \end{array} \right]$$

Parametric solution:

$x = -29 + 5z$
$y = 53 - 6z$
$z = any\ number$

Once again we can not buy a part of a tank car. So z must be an integer. We also know that all of the variables must be greater than or equal to zero.

$$
\begin{array}{ccc}
x \geq 0 & y \geq 0 & z \geq 0 \\
-29 + 5z \geq 0 & 53 - 6z \geq 0 & \\
5z \geq 29 & 53 \geq 6z & \\
z \geq 5.8 & 53/6 \geq z & \\
& z \leq \frac{53}{6} \approx 8.8333 &
\end{array}
$$

In addition we know that the variables can not be any larger than 24

$$
\begin{array}{ccc}
x \leq 24 & y \leq 24 & z \leq 24 \\
-29 + 5z \leq 24 & 53 - 6z \leq 24 & \\
5z \leq 53 & 29 \leq 6z & \\
z \leq 10.6 & 29/6 \leq z & \\
& z \geq \frac{29}{6} \approx 4.8333 &
\end{array}
$$

Taken all together, we find that $z = 6$, 7, 8.

This problem ends up having only three solutions.