Solve these systems of equations

All of these matrices can not be solved by the rref command on the TI-84 as they are written.

1. Solve for x, y, and z

$$\begin{aligned} 3x + 6y - 9z &= 15 \\ 2x + 4y - 6z &= 10 \\ -2x - 3y + 4z &= -6 \\ 3x + 7y - 11z &= 19 \\ 4x + 9y - 14z &= 24 \end{aligned}$$

2. Solve for x, y, and z

$$\begin{aligned} 3x - 2y + z &= -7 \\ 2x + y - 4z &= 0 \\ x + y - 3z &= 1 \\ 7x - 3z &= -7 \\ 7x + y - 9z &= 5 \end{aligned}$$

3. Solve for x, y, and z

$$\begin{aligned} 2x - 4y - z &= -8 \\ 4x - 8y + 3z &= 4 \\ -2x + 4y + z &= 11 \\ 3x + 2y - 4z &= 6 \\ x + 2y + 4z &= -16 \end{aligned}$$

The next three problems are solved by the Gauss-Jordan method.

1. Solve for x, y, and z

$$\begin{aligned} 3x + 6y - 9z &= 15 \\ 2x + 4y - 6z &= 10 \\ -2x - 3y + 4z &= -6 \\ 3x + 7y - 11z &= 19 \\ 4x + 9y - 14z &= 24 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 3 & 6 & -9 & 15 \\ 2 & 4 & -6 & 10 \\ -2 & -3 & 4 & -6 \\ 3 & 7 & -11 & 19 \\ 4 & 9 & 14 & 24 \end{array}\right] \quad R_1(\frac{1}{3}) \quad \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 2 & 4 & -6 & 10 \\ -2 & -3 & 4 & -6 \\ 3 & 7 & -11 & 19 \\ 4 & 9 & 14 & 24 \end{array}\right] \begin{array}{l} \\ R_2 + (-2)R_1 \\ R_3 + (2)R_1 \\ R_4 + (-3)R_1 \\ R_5 + (-4)R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 4 \\ 0 & 1 & -2 & 4 \\ 0 & 1 & -2 & 4 \end{array}\right] \quad R_2 \leftrightarrow R_3 \quad \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 4 \\ 0 & 1 & -2 & 4 \end{array}\right] \begin{array}{l} R_1 + (-2)R_2 \\ \\ \\ R_4 + (-1)R_2 \\ R_5 + (-1)R_2 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & -3 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Parametric Solution
x = -3 - z
y = 4 + 2z
z = any number

2. Solve for x, y, and z

$$\begin{aligned} 3x - 2y + z &= -7 \\ 2x + y - 4z &= 0 \\ x + y - 3z &= 1 \\ 7x - 3z &= -7 \\ 7x + y - 9z &= 5 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & -7 \\ 2 & 1 & -4 & 0 \\ 1 & 1 & -3 & 1 \\ 7 & 0 & -3 & -7 \\ 7 & 1 & -9 & 5 \end{array}\right] \quad R_1 \leftrightarrow R_3 \quad \left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 2 & 1 & -4 & 0 \\ 3 & -2 & 1 & -7 \\ 7 & 0 & -3 & -7 \\ 7 & 1 & -9 & 5 \end{array}\right] \begin{array}{l} \\ R_2 + (-2)R_1 \\ R_3 + (-3)R_1 \\ R_4 + (-7)R_1 \\ R_5 + (-7)R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & -1 & 2 & -2 \\ 0 & -5 & 10 & -10 \\ 0 & -7 & 18 & -14 \\ 0 & -6 & 12 & -2 \end{array}\right] \quad R_2(-1) \quad \left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & -5 & 10 & -10 \\ 0 & -7 & 18 & -14 \\ 0 & -6 & 12 & -2 \end{array}\right] \begin{array}{l} R_1 + (-1)R_2 \\ \\ R_3 + (5)R_2 \\ R_4 + (7)R_2 \\ R_5 + (6)R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 10 \end{array}\right]$$

The last row of the matrix say that system of equations has no solution.

3. Solve for x, y, and z

$$\begin{array}{rcr} 2x - 4y - z & = & -8 \\ 4x - 8y + 3z & = & 4 \\ -2x + 4y + z & = & 11 \\ 3x + 2y - 4z & = & 6 \\ x + 2y + 4z & = & -16 \end{array} \quad \left[\begin{array}{ccc|c} 2 & -4 & -1 & -8 \\ 4 & -8 & 3 & 4 \\ -2 & 4 & 1 & 11 \\ 3 & 2 & -4 & -6 \\ 1 & 2 & 4 & -16 \end{array}\right] \quad R_1 \leftrightarrow R_5 \quad \left[\begin{array}{ccc|c} 1 & 2 & 4 & -16 \\ 4 & -8 & 3 & 4 \\ -2 & 4 & 1 & 11 \\ 3 & 2 & -4 & -6 \\ 2 & -4 & -1 & -8 \end{array}\right] \begin{array}{l} \\ R_2 + (-4)R_1 \\ R_3 + (2)R_1 \\ R_4 + (-3)R_1 \\ R_5 + (-2)R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & -16 \\ 0 & -16 & -13 & 68 \\ 0 & 8 & 9 & -21 \\ 0 & -4 & -16 & 54 \\ 0 & -8 & -9 & 24 \end{array}\right] \quad R_2(\frac{-1}{16}) \quad \left[\begin{array}{ccc|c} 1 & 2 & 4 & -16 \\ 0 & 1 & \frac{13}{16} & \frac{-17}{4} \\ 0 & 8 & 9 & -21 \\ 0 & -4 & -16 & 54 \\ 0 & -8 & -9 & 24 \end{array}\right] \begin{array}{l} R_1 + (-2)R_2 \\ \\ R_3 + (-8)R_2 \\ R_4 + (4)R_2 \\ R_5 + (8)R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{19}{8} & \frac{-15}{2} \\ 0 & 1 & \frac{13}{16} & \frac{-17}{4} \\ 0 & 0 & \frac{5}{2} & 13 \\ 0 & 0 & \frac{-51}{4} & 37 \\ 0 & 0 & \frac{-5}{2} & -10 \end{array}\right] \quad R_3(\frac{2}{5}) \quad \left[\begin{array}{ccc|c} 1 & 0 & \frac{19}{8} & \frac{-15}{2} \\ 0 & 1 & \frac{13}{16} & \frac{-17}{4} \\ 0 & 0 & 1 & \frac{26}{5} \\ 0 & 0 & \frac{-51}{4} & 37 \\ 0 & 0 & \frac{-5}{2} & -10 \end{array}\right] \begin{array}{l} R_1 + (\frac{-19}{8})R_3 \\ R_2 + (\frac{-13}{16})R_3 \\ \\ R_4 + (\frac{51}{4})R_3 \\ R_5 + (\frac{5}{2})R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-397}{20} \\ 0 & 1 & 0 & \frac{-339}{40} \\ 0 & 0 & 1 & \frac{26}{5} \\ 0 & 0 & 0 & \frac{1033}{10} \\ 0 & 0 & 0 & 3 \end{array}\right]$$

Both of the last rows show that this system of equations does not have a solution.

NOTE: You can also conclude that this system of equations doesn't have a solution by looking at the third augmented matrix and comparing rows three and five. 8y + 9z can not equal both -21 and -24.