

Solutions to Sample Problems for Exam 3

1. (a) \$412.50  
(b) 16%
2. (a) \$992.65  
(b) \$92.65
3. \$6496.46
4. \$321.26
5. (a) \$20,422.84  
(b) \$3,422.84  
(c) \$674.96
6. \$169.92
7. (a) \$52,290.48  
(b) \$25,290.48  
(c) 58  
(d) \$1,218.75
8. (a) \$1,471.03  
(b) \$94,239.75  
(c) amortization chart

period	interest owed	payment	amt. toward principal	outstanding principal
0	—	—	—	184,800
1	785.40	1471.03	685.63	184,114.37

9. There is more than one answer for this problem.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 5 & 8 \end{array} \right]$$

10. There is more than one answer for this problem.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

11. There is more than one answer for this problem.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 8 \end{array} \right]$$

12. (a) **I)**  $x$  = the amount invested in high-risk stocks.  
 $y$  = the amount invested in medium-risk stocks.  
 $z$  = the amount invested in low-risk stocks.

**II)**  $x + y + z = 300,000$   
 $.16x + .10y + .04z = 33,000$   
 $2x - y + 2z = 0$

**III)**  $x = \$75,000$ ,  $y = \$200,000$ , and  $z = \$25,000$

- (b) **I)**  $x$  = number of tank cars purchased with 6,000 gallon capacity  
 $y$  = number of tank cars purchased with 8,000 gallon capacity  
 $z$  = number of tank cars purchased with 18,000 gallon capacity

**II)**  $x + y + z = 24$   
 $6000x + 8000y + 18000z = 250000$

**III)** Parametric solution:

$x = -29 + 5z$   
 $y = 53 - 6z$   
 $z = \text{any number}$

now to place restrictions on the parameter.  
 Since the number of cars has to be greater than or equal to zero.

$$\begin{array}{lll} x \geq 0 & y \geq 0 & z \geq 0 \\ -29 + 5z \geq 0 & 53 - 6z \geq 0 & \\ 5z \geq 29 & 53 \geq 6z & \\ z \geq 5.8 & 8.83333 \geq z & \\ & z \leq 8.83333 & \end{array}$$

Since the number of cars has to be less than or equal to 24.

$$\begin{array}{lll} x \leq 24 & y \leq 24 & z \leq 24 \\ -29 + 5z \leq 24 & 53 - 6z \leq 24 & \\ 5z \leq 53 & 29 \leq 6z & \\ z \leq 10.6 & 4.83333 \leq z & \\ & z \geq 4.83333 & \end{array}$$

Now using all of the above information at the same time, we see that  $5.8 \leq z \leq 8.83333$ . Since we can not buy a part of a tank car,  $z$  must be an integer so the only values of  $z$  that work are 6, 7, 8.

13.  $\left[ \begin{array}{ccc|c} 3 & 6 & 15 & 9 \\ 7 & 12 & 39 & 25 \\ 2 & 6 & 5 & 4 \\ 3 & 0 & 6 & 1 \end{array} \right] R_1(\frac{1}{3}) \rightarrow R_1$

$$\left[ \begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 7 & 12 & 39 & 25 \\ 2 & 6 & 5 & 4 \\ 3 & 0 & 6 & 1 \end{array} \right] \quad \begin{array}{l} R_2 + (-7)R_1 \rightarrow R_2 \\ 3R_3 + (-2)R_4 \rightarrow R_3 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & -2 & 4 & 4 \\ 0 & 18 & 3 & 10 \\ 3 & 0 & 6 & 1 \end{array} \right]$$

14.  $x = 20$ ,  $y = -11$ ,  $u = 5$ , and  $z = -2$

15.  $K = \begin{bmatrix} 7 & -8 & 5 \\ -24.5 & 27 & -8.5 \\ 105 & -100 & 19 \end{bmatrix}$

16. (a)  $6 + 3y$   
(b)  $3x + 12$

17.  $D + C =$  not possible: not same dim.

$$D - 3B = \begin{bmatrix} -2 & 1 & -9 \\ -1 & -3 & -1 \end{bmatrix}$$

$$DC = \begin{bmatrix} 1 & -6 \\ 7 & 6 \end{bmatrix}$$

$DA =$  not possible: the number of rows in A is not equal to the number of cols. in D.

$$B + C^T = \begin{bmatrix} 2 & -1 & 7 \\ -2 & 4 & 0 \end{bmatrix}$$

$B^{-1}$  not possible B is not square.

$$A^{-1} = \begin{bmatrix} 1 & 0 \\ -0.5 & -0.5 \end{bmatrix}$$

$E^{-1}$  not possible, singular matrix.

18. Each number represents the hourly rate for each crew. John's crew has an hourly rate of \$68.05 and Matt's crew has an hourly rate of \$60.10.

19. (a)  $x = -14$ ,  $y = 39$ ,  $z = -9$   
(b)  $x = -12$ ,  $y = 37$ ,  $z = -10$