

1. The height, measured in feet, of a certain type of tree is proportional to the square-root of the circumference, measured in inches. It has been noted that a tree with a circumference of 26 inches has a height of 50 feet.

(a) What is the constant of proportionality?

$$H = K\sqrt{C}$$

$$50 = K\sqrt{26}$$

$$\frac{50}{\sqrt{26}} = K \approx 9.805806757$$

(b) If a tree has a circumference of 380 inches, what would be the height of the tree?

$$H = \frac{50}{\sqrt{26}} \cdot \sqrt{380} = 191.15036ft$$

2. Suppose  $f$ ,  $g$ , and  $h$  are functions of  $x$  such that one of them is proportional to  $x$ , one is inversely proportional to  $x$ , and one is proportional to the square of  $x$ . Using the table below, write  $f$ ,  $g$ , and  $h$  as functions of  $x$  and find the constants of proportionality:

$x$	$f(x)$	$g(x)$	$h(x)$
5	600	50	1.25
10	300	200	2.50
15	200	450	3.75
20	150	800	5.00
25	120	1250	6.25

$$600 = \frac{k}{5}$$

$$3000 = k$$

$$y = \frac{3000}{x}$$

$$f(x) = \frac{3000}{x}$$

$$y = kx$$

$$y = \frac{k}{x}$$

$$y = kx^2$$

$$50 = k \cdot 5$$

$$10 = k$$

$$y = 10x$$

doesn't work  
for  $g$ .

$$h = kx$$

$$1.25 = k \cdot 5$$

$$.25 = k$$

$$50 = k \cdot 5^2$$

$$2 = \frac{50}{25} = k$$

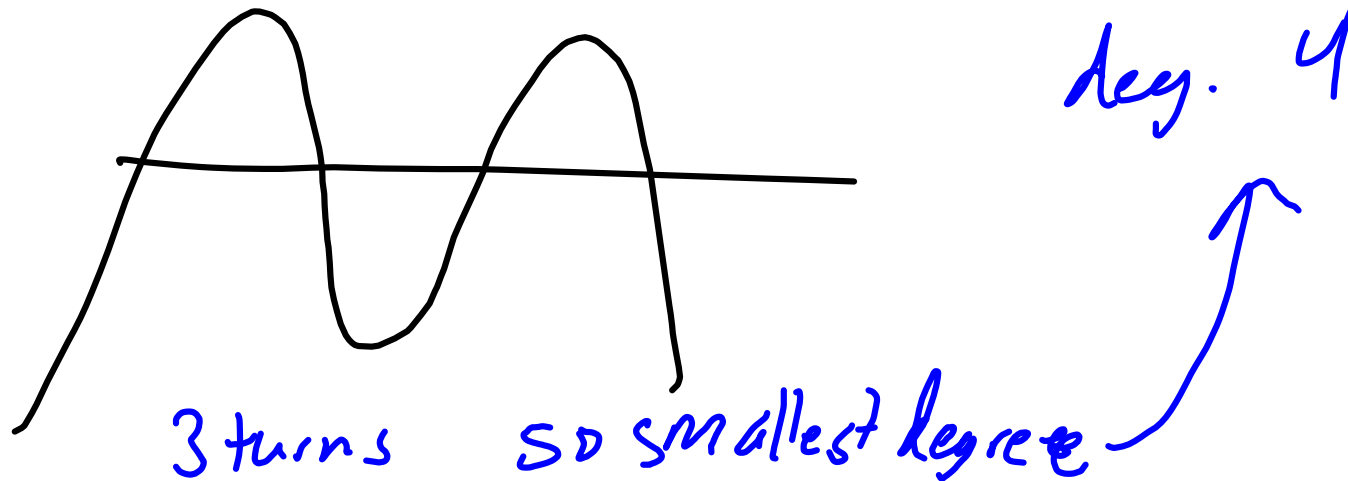
$$y = 2x^2$$

$$g(x) = 2x^2$$

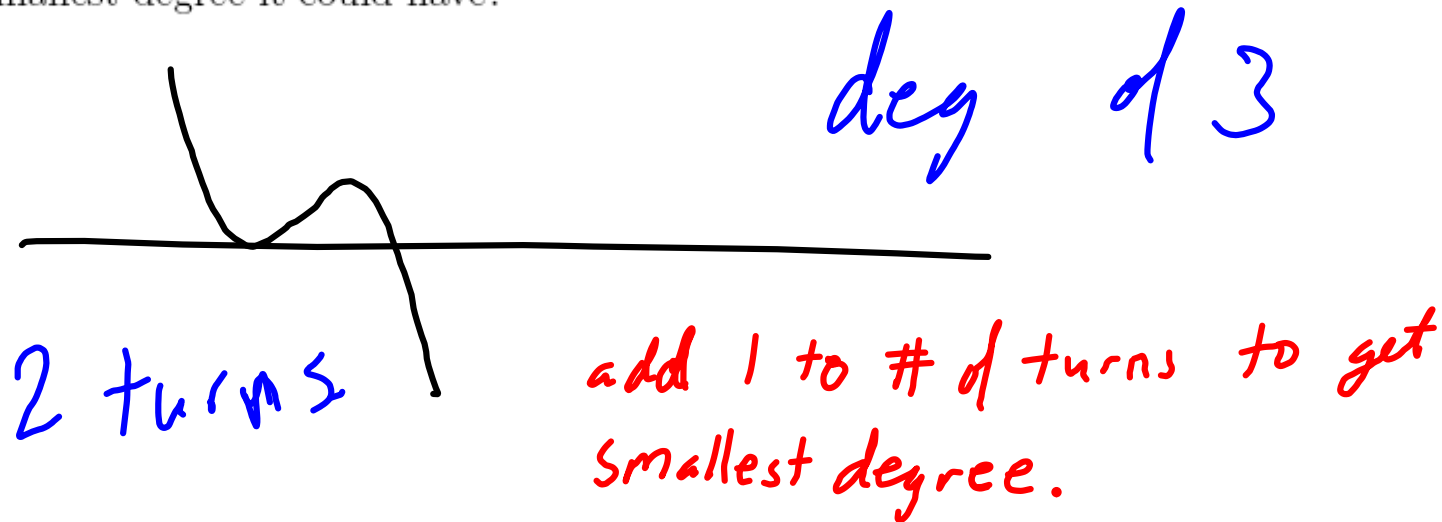
$$h(x) = .25x$$

Alternate method use power Regression

3. A polynomial crosses the horizontal axis 4 times. What is the smallest degree it can have?



4. A polynomial touches the horizontal axis twice but only crosses it once. What is the smallest degree it could have?



$$y = A \cos(Bx) + C \quad y = A \sin(Bx) + C$$

5. The height of water in a harbor was given by the formula  $h(t) = 4.9 + 4.4 \cos\left(\frac{\pi t}{6}\right)$  where  $h$  is measured in feet and  $t$  is measured in hours since midnight.

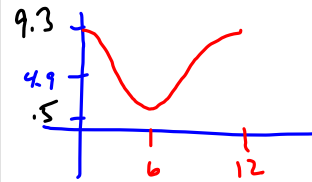
(a) Find the amplitude.  $4.4 \text{ ft}$

$$\text{period} = \frac{2\pi}{8}$$

(b) Find the period.

$$\frac{2\pi}{\frac{\pi}{6}} = 2\pi \cdot \frac{6}{\pi} = 12 \text{ hours.}$$

(c) Find the lowest level that the water can have and give the first time this happened (as measured from midnight).



$\frac{1}{2} \text{ ft}$  6 hrs  
after midnight.

$\frac{1}{2} \text{ ft}$  at 6 AM

(d) Evaluate  $h(8)$  and interpret the value.

$$h(8) = 4.9 + 4.4 \cos\left(\frac{\pi}{6} \cdot 8\right) = 2.7 \text{ ft.}$$

at 8 AM the height of the water was 2.7 ft

(e) Compute the average rate of change from midnight to 8 am. Interpret this answer.

$$\frac{h(8) - h(0)}{8 - 0} = \frac{2.7 - 9.3}{8 - 0} = -.825 \text{ ft/hr}$$

from midnight to 8 AM the height of the water dropped on avg. by .825 ft each hour.

6. Find the amplitude and the period of  $y = 7 + 2 \sin(4t)$

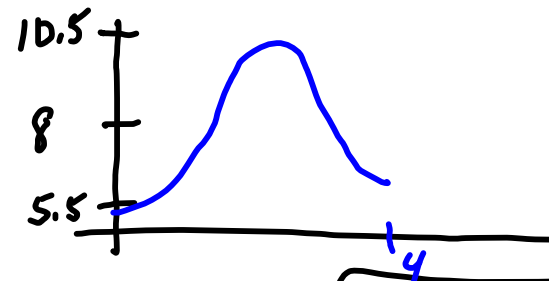
$$\text{amp.} = 2$$

$$\text{Period} = \frac{2\pi}{4} = \frac{\pi}{2}$$

7. The depth of water in a tank oscillates once every 4 hours around an average depth of 8 feet. If the smallest depth is 5.5 feet, find a formula for the depth in terms of time(hours)

(a) Assume the water starts at its lowest level.

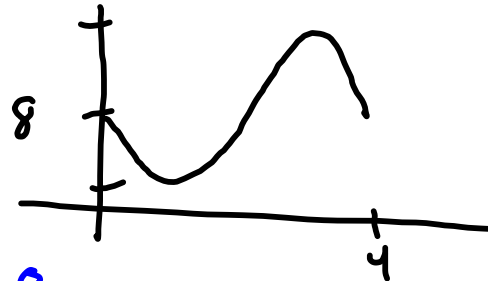
$$-2.5 \cos\left(\frac{\pi}{2}x\right) + 8 = y$$



$$\begin{aligned} \text{Amp} &= 2.5 \\ \text{period} &= 4 \end{aligned}$$

(b) Assume the water starts at its average level and is dropping.

$$y = -2.5 \sin\left(\frac{\pi}{2}x\right) + 8$$



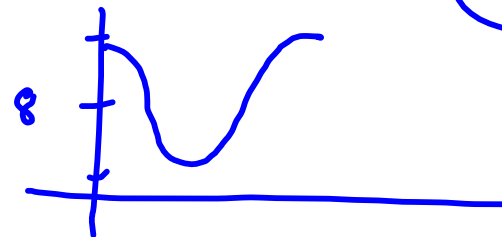
$$\text{period} = \frac{2\pi}{B}$$

$$4 = \frac{2\pi}{B}$$

$$B = \frac{2\pi}{4} = \frac{\pi}{2}$$

(c) Assume the water starts at its highest level.

$$y = 2.5 \cos\left(\frac{\pi}{2}x\right) + 8$$



8. The table gives the population of a country in millions.

Year	1790	1830	1870	1910	1950	1990
population	2	5.5	12.3	25.9	38.5	50.8

formula is nicer with time shift of time starts in 61.17371  
 $y = \frac{61.17371}{1 + 26.1923e^{-.02409x}}$

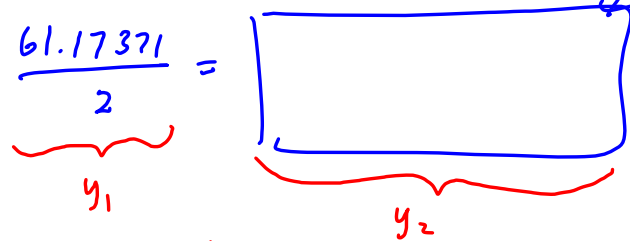
(a) Find the best fitting formula for the data.

$$y = \frac{61.17371}{1 + (1.4011 \times 10^{20})e^{-.0240915x}}$$

(b) Estimate the limiting population.

61.17371 million

(c) In what year does the function change from concave up to concave down?



used the calculator to find the intersection

1925      2<sup>nd</sup> Calc choice #5

(d) Find and interpret the average rate of change from 1830 to 1910.

$$\frac{25.9 - 5.5}{1910 - 1830} = .255 \text{ million/yr}$$

from 1830 to 1910 the population of the country increased on average by .255 million people each year.

9. The table lists the time in seconds that an average athlete takes to swim 100 meters freestyle at a certain age.

Age	8	12	16	20	24	28	32
time	92	70	58	51	49	53	60

Find the best fitting formula for the data. Justify your answer.

Look at scatter plot. it is definitely not  
exp, log, linear,  
or power.  
→ quad. looks best.

$$y = .17485x^2 - 8.235119x + 145.42857$$



10. Let  $f(t) = t^2$  and  $g(t) = 4te^t$ . Find

(a)  $g(f(x))$

$$g(x^2) = 4x^2 e^{x^2}$$

$$f(x) = x^2$$

(b)  $f(g(x))$

$$g(x) = 4xe^x$$

$$f(4xe^x) = (4xe^x)^2$$

$$= 16x^2 e^x e^x$$

$$= 16x^2 e^{2x}$$

11. Describe how the graph of  $2f(x - 3) + 6$  differs from the graph of  $f(x)$ .

1) Right 3 units

2) stretch by a factor of 2  
ie multiply by 2.

3) shift up 6 units.

12. solve for x.  $Ae^{1.5x} = 7e^{2.4x}$

$$\ln(Ae^{1.5x}) = \ln(7e^{2.4x})$$

$$\ln(A) + \cancel{\ln e^{1.5x}} = \ln(7) + \cancel{\ln e^{2.4x}}$$

$$\ln(A) + 1.5x = \ln(7) + 2.4x$$

$$\ln(A) - \ln(7) = 2.4x - 1.5x$$

$$\ln(A) - \ln(7) = .9x$$

$$\boxed{\frac{\ln(A) - \ln(7)}{.9} = x}$$

13. Your parents have started a bank account for you with \$6,000. The bank account earns interest at a rate of 5% compounded continuously. At the end of five years you make a one-time withdrawal of \$2500. Find the times where the balance of the account was \$7000. Give your answers from the start of the account.

$$y = 6000 e^{.05x}$$

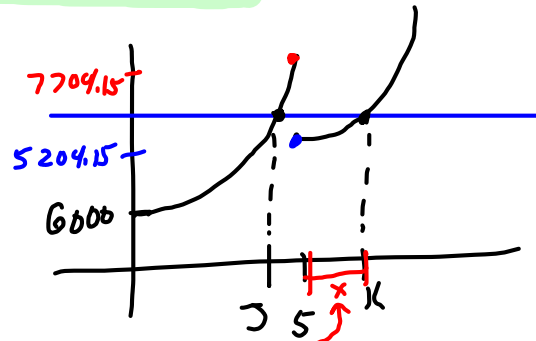
Solve for J.

$$7000 = 6000 e^{.05x}$$

$$\frac{7}{6} = e^{.05x}$$

$$\ln\left(\frac{7}{6}\right) = \ln e^{.05x} = .05x$$

$$\frac{\ln\left(\frac{7}{6}\right)}{.05} = x \approx 3.0830 \text{ yrs}$$



Solve for K.

$$7000 = 5204.15 e^{.05x}$$

$$\frac{7000}{5204.15} = e^{.05x}$$

$$\ln\left(\frac{7000}{5204.15}\right) = .05x$$

$$x = \frac{\ln\left(\frac{7000}{5204.15}\right)}{.05}$$

$$x = 5.9291 \text{ yrs}$$

+5

$$10.9291 \text{ yrs}$$

14. A chemical has a continuous rate of decay of 14% and time is given in years.

(a) What is the relative rate of decay?

$$k = -.14$$

$$a = e^k$$

$$a = \frac{e^{-.14} = .869358}{-.13064176}$$

Rel rate of decay  
13.064176%

(b) How long until 37% of the chemical is gone?

$$63 = 100 e^{-.14x}$$

$$.63 = e^{-.14x}$$

$$\ln(.63) = -.14x$$

$$\frac{\ln(.63)}{-.14} = x = 3.3003 \text{ yrs.}$$

Can also use  
the calc +  
graph.

15. A bee colony has 500 worker bees. If the colony is growing at a rate of 4% per week.

- (a) What is the continuous growth rate.  
(b) How long does it take the number of workers to triple?

$$a = 1 + .04 = 1.04$$

$$a = e^k$$

$$\ln(a) = k$$

cont. growth rate  
of 3.92207%

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$$\underbrace{1500}_{y_1} = \underbrace{500}_{y_2} (1.04)^x$$

used calc  $x = 28.011023$  weeks

16. A radioactive chemical was spilled and the radiation level was 2.8 millirem/hour, which is 5 times the safe level recommended by the government. After three hours the radiation level was 2.629 millirem/hour.

- (a) Find the formula,  $f(x)$ , that gives the radiation level as a function of time when the radiation level is decaying at a continuous rate.

$$\left. \begin{array}{l} (0, 2.8) \\ (3, 2.629) \end{array} \right\} \text{exp reg } f(x) = y = 2.8 (.979213784)^x$$

- (b) Compute  $f(4)$  and interpret the answer.

$$f(4) = 2.57435 \text{ millirem/hr}$$

4 hrs after the chemical was spilled  
the radiation level was 2.57435 millirem/hr

- (c) How long until it is safe for people to return to the area?

$$\frac{2.8}{5} = 2.8 (.979213784)^x$$

used the calc.

76.62059 hrs

- (d) What is the half-life of the chemical?

note  $\frac{1}{2}$ -Life doesn't  
depend on the  
initial size

$$.5 = 1 (.979213784)^x$$

$$\ln(.5) = \ln (.979213784)^x$$

$$\frac{\ln(.5)}{\ln(.979213784)} = x \approx 32.99869 \text{ hrs}$$

17. Find the equation of the line through the point  $(7, 15)$  and has an average rate of change of 8 from  $x = 4$  to  $x = 9$ .

$$y - 15 = 8(x - 7)$$

$$y - 15 = 8x - 56$$

$$y = 8x - 41$$

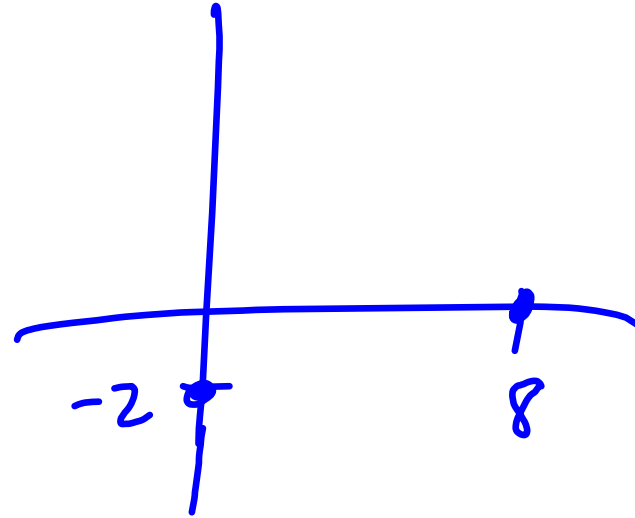


18. Find the equation of a line with a horizontal intercept of 8 and a vertical intercept of -2.

$$(8, 0)$$
$$(0, -2)$$

$$m = \frac{-2 - 0}{0 - 8} = \frac{-2}{-8} = \frac{1}{4}$$

$$y = \frac{1}{4}x - 2$$



$$y - (-2) = \frac{1}{4}(x - 0)$$

$$y + 2 = \frac{1}{4}x$$

$$y = \frac{1}{4}x - 2$$

19. The monthly charge for a waste collection service is \$32 for 100kg of waste and \$48 for 180 kg of waste.

(a) Find a linear formula for the cost as a function of the number of kilograms of waste.

$(w, c)$

$(100, 32)$

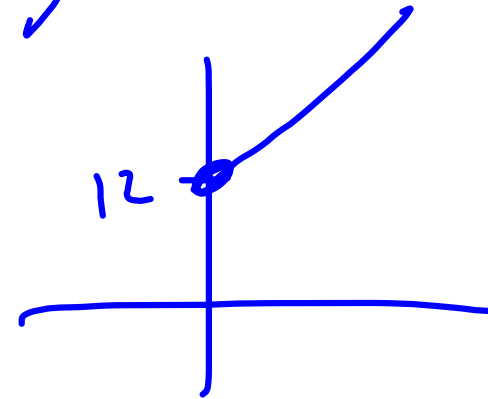
$(180, 48)$

$$\text{cost} = .2x + 12$$

with lin Reg.

(b) Find the vertical intercept and interpret this value.

vert. int. = 12



Basic charge is \$12 per month ..