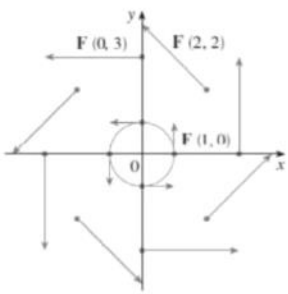
Instructor: Rosanna Pearlstein

Math 251 – Spring 2023
"Week-in-Review"

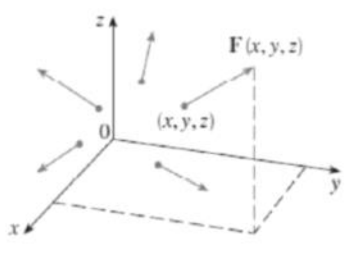
Wir 9: Sections 16.1, 16.2, 16.3.

Section 16.1

Definition: A **vector field** in two dimension is a function $\mathbf{F}$ that assigns to each point (x, y) in $D \subset \mathbb{R}^2$ a two dimensional vector, $\mathbf{F}(x, y)$.
In two dimension, the vector field lies entirely in the xy plane.
Here is a vector field in $\mathbb{R}^2$:



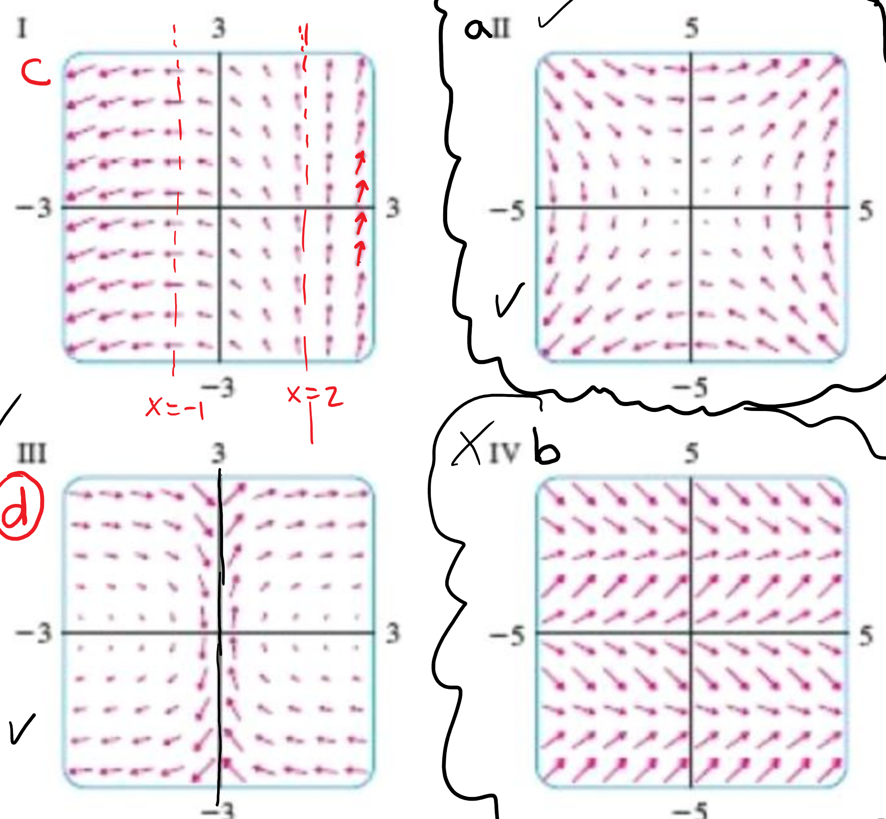
Definition: A **vector field** in three dimension is a function $\mathbf{F}$ that assigns to each point (x, y, z) in $D \subset \mathbb{R}^3$ a three dimensional vector, $\mathbf{F}(x, y, z)$.
In three dimension, the vector field is in space.
Here is a vector field in $\mathbb{R}^3$:



In order to match $\mathbf{F}$ with its vector field, choose a several points, (x, y), in each quadrant, and look at the *direction* of $\mathbf{F}(x, y)$. To narrow down further, look at the behavior of the components. Often times, it is a process of elimination.

Problem 1. Match each vector field equation with its graph:

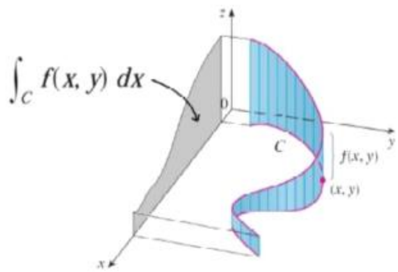
a) $\boldsymbol{F}(x,y) = \langle y, 0 \rangle$ – Q I
b) $\boldsymbol{F}(x,y) = \langle 1, \sin y \rangle$
c) $\boldsymbol{F}(x,y) = \langle x-2, x+1 \rangle$
d) $\boldsymbol{F}(x,y) = \langle y, \frac{1}{x} \rangle$



Section 16.2

Definition: If f is defined on a smooth curve C defined as $\mathbf{r}(t) = \langle x(t), y(t) \rangle$, $a \le t \le b$, then the **line integral of f along C** is

$$\int_C f(x,y)ds = \int_a^b (f(x(t), y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_a^b (f(x(t), y(t))|\mathbf{r}'(t)|\, dt$$



need a parametrization

In order to find a line integral along a curce C, we must first parameteterize the curve. Sometimes, the parameterization will be given explicitly, other times you must parameterize the curve.

set up first

$ds = \sqrt{x'(t)^2 + y'(t)^2}$

Problem 2. Evaluate $\int_C (2x + y)ds$, where C is defined as $\mathbf{r}(t) = \langle \overset{x}{2 + t}, \overset{y}{3 - t} \rangle$, $0 \le t \le 1$.

$x'(t) = 1$ $\quad y'(t) = -1$

$$\int_0^1 \left[2(2+t) + 3 - t\right] \sqrt{1^2 + (-1)^2}\, dt =$$

$4 + 2t + 3 - t$

$$\sqrt{2} \int_0^1 (7 + t)\, dt =$$

$$\sqrt{2}\left[7 + \frac{1}{2}\right] = \sqrt{2}\, \frac{15}{2}$$

MATHEMATICS TEXAS A&M UNIVERSITY

Instructor: Rosanna Pearlstein

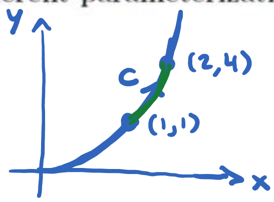
Math Learning Center

Math 251 – Spring 2023
“Week-in-Review”

Problem 3. Set up but do not evaluate $\int_C (2x + x^2 y)\,ds$, where C is the arc of the curve $y = x^2$ from $(1,1)$ to $(2,4)$ using two different parameterizations.

$y = x^2$

$\sqrt{y} = x$



Sol 1: $\vec{r}(t) = \langle \overset{x}{t}, \overset{y}{t^2} \rangle \qquad 1 \le t \le 2$

$\sec^2 x = 1 + \tan^2 x$

$$\int_1^2 (2t + t^2 t^2)\sqrt{(1)^2 + (2t)^2}\,dt = \int_1^2 (2t + t^4)\sqrt{1 + 4t^2}\,dt$$

$u = 1 + t^2$, $du = 2t\,dt$

$$\int_1^2 2t\sqrt{1 + t^2}\,dt$$

$$\int t^4 \sqrt{1 + 4t^2}\,dt$$

$u = 1 + 4t^2$

$t^2 = \frac{u-1}{4}$

$\int$

2) $\vec{r}(t) = \langle \sqrt{t}, t \rangle$

$$\int_1^4 (2\sqrt{t} + t\,t)\sqrt{\left(\frac{1}{2\sqrt{t}}\right)^2 + 1^2}\,dt$$

$$\int_1^4 (2\sqrt{t} + t^2)\sqrt{\frac{1}{4t} + 1}\,dt$$

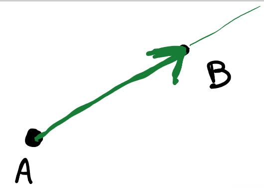
$\sqrt{\frac{1 + 4t}{4t}}$

$1 + \frac{1}{4t} = u$

$\vec{AB}$

$\overrightarrow{AB}$

$A + \underbrace{3t}_{?}\overrightarrow{AB}$ $\quad 0 \le t \le \frac{1}{3}$



Problem 4. Evaluate $\int_C (x^2 + y)\, ds$ where C consists of the line segment from the point $(1, 4)$ to $(3, -1)$.

$\vec{r}(t) = \langle 1, 4 \rangle + t\left[\langle 3, -1 \rangle - \langle 1, 4 \rangle\right] =$

$= \langle 1, 4 \rangle + t \langle 2, -5 \rangle =$ $\quad 0 \le t \le 1$

$\vec{r}(t) = \langle \underbrace{1 + 2t}_{x}, \underbrace{4 - 5t}_{y} \rangle$

$$\int_0^1 \left[(1 + 2t)^2 + 4 - 5t\right] \sqrt{2^2 + (-5)^2}\, dt$$

$1 + 4t + 4t^2 + 4 - 5t$

$$\sqrt{29} \int_0^1 (5 - t + 4t^2)\, dt =$$

$$\sqrt{29}\left[5 - \frac{1}{2} + \frac{4}{3}\right] = \sqrt{29}\left(\frac{30 - 3 + 8}{6}\right) = \sqrt{29}\,\frac{35}{6}$$

MATHEMATICS
TEXAS A&M UNIVERSITY

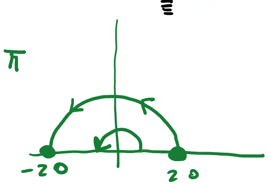
Instructor: Rosanna Pearlstein

Math Learning Center

Math 251 – Spring 2023
"Week-in-Review"

Problem 5. Evaluate $\int_C (x+y)ds$, where C is the top half of the circle $x^2+y^2=4$, oriented counterclockwise.

$\vec{r}(t) = \langle 2\cos t,\ 2\sin t \rangle \qquad 0 \le t \le \pi$

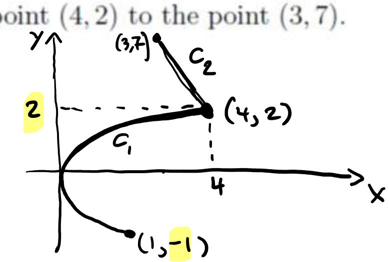


$$\int_0^{\pi} 2(\cos t + \sin t)\underbrace{\sqrt{(-2\sin t)^2+(2\cos t)^2}}_{=\sqrt{4(\sin^2+\cos^2)}\,=\,2}\,dt =$$

$$= \int_0^{\pi} 4(\cos t + \sin t)\,dt = 4\Big[\overbrace{\sin t}^{\text{zero}} - \cos t\Big]_0^{\pi} =$$

$$= 4\left[--1--1\right] = 8$$

Problem 6. Set up but do not evaluate $\int_C (2 + x^2 y)\,ds$, where C is the arc of the curve $\boxed{x = y^2}$ from $(1, -1)$ to $(4, 2)$ and then along the line segment from the point $(4, 2)$ to the point $(3, 7)$.



$\vec{r}_1 = \langle t^2, t \rangle \qquad -1 \le t \le 2$

$\vec{r}_2 = \langle 4, 2 \rangle + t\left[\langle 3,7 \rangle - \langle 4, 2 \rangle\right]$

$= \langle 4 - t, 2 + 5t \rangle \qquad 0 \le t \le 1$

$$\int_C = \int_{C_1} + \int_{C_2}$$

$$(C_1)\int_{-1}^{2} \left[2 + (t^2)^2 t\right] \sqrt{(2t)^2 + 1^2}\,dt + (C_2)\int_{0}^{1} \left[2 + (4-t)^2 (2+5t)\right] \sqrt{(-1)^2 + 5^2}\,dt$$

MATHEMATICS
TEXAS A&M UNIVERSITY

Math Learning Center

Instructor: Rosanna Pearlstein

Math 251 – Spring 2023
"Week-in-Review"

Line Integrals ~~**over**~~ of **vector fields:** Suppose now are moving a particle along a curve C through a vector (force) field, $\mathbf{F}$. We define the **line integral of F along C** to be

$$\text{Work} = \int_c \mathbf{F}\cdot d\mathbf{r} = \int_a^b (\mathbf{F}(\mathbf{r}(t)) \bullet \mathbf{r}'(t)\,dt$$

need a parametrization

Problem 7. Find $\int_c \mathbf{F}\cdot\mathbf{r}$, where C is defined by $\boxed{\mathbf{r}(\mathbf{t}) = \langle t, t^2, t^4\rangle}$, $0 \le t \le 1$, and $\mathbf{F}(x, y, z) = \langle x, z^2, -4y\rangle$.

$$\int_0^1 \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

$\vec{F}(\vec{r}(t)) = \langle t, t^8, -4t^2\rangle$

$\vec{r}\,'(t) = \langle 1, 2t, 4t^3\rangle$

$\vec{F}\cdot\vec{r}\,' = t + 2t^9 - 16t^5$

$$\int_0^1 (t + 2t^9 - 16t^5)\,dt =$$

$$= \frac{1}{2} + \frac{2}{10} - \frac{16}{6} = \frac{1}{2} + \frac{1}{5} - \frac{8}{3} = \frac{15+6-80}{30} = -\frac{59}{30}$$

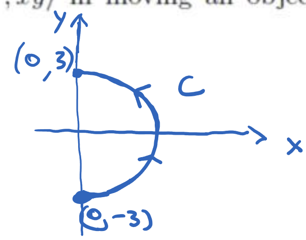
MATHEMATICS
TEXAS A&M UNIVERSITY

Math Learning Center

Instructor: Rosanna Pearlstein

Math 251 – Spring 2023
"Week-in-Review"

Problem 8. Find the work done by the force field $\mathbf{F}(x, y) = \langle x^2, xy\rangle$ in moving an object counterclockwise around the right half of the circle $x^2 + y^2 = 9$.



$\vec{r}(t) = < 3\cos t, 3\sin t >$ $\quad -\frac{\pi}{2} \le t \le +\frac{\pi}{2}$

$\int_C \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt =$

$= \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} 0 \, dt = 0$

$\vec{F}(\vec{r}(t)) = < 3^2 \cos^2 t, 3 \cdot 3 \cos t \sin t >$

$\vec{r}\,'(t) = <-3\sin t, \ 3\cos t>$

$\vec{F} \cdot \vec{r}\,' = -27\cos^2 t \sin t + 27 \cos^2 t \sin t =$

$= 0$

Instructor: Rosanna Pearlstein

Math Learning Center

Math 251 – Spring 2023
"Week-in-Review"

Definition: Let C be a smooth curve defined by the parametric equations $x = x(t)$, $y = y(t)$ for $a \le t \le b$. **The line integral of f along** C **with respect to** x **is** $\int_C f(x,y)dx = \int_a^b (f(x(t), y(t))\ x'(t)\ dt.$

The line integral of f along C **with respect to** y **is** $\int_C f(x,y)dy = \int_a^b (f(x(t), y(t))\ y'(t)\ dt$

Problem 9. $\int_C y dx + x^2 dy$, where C is decribed by $\mathbf{r}(t) = \langle 3e^t, e^{2t} \rangle$, $0 \le t \le 1$.

x y

$dx = 3e^t dt$

$dy = 2e^{2t} dt$

$\int \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = \langle y, x^2 \rangle$

$$\int_0^1 e^{2t}(3e^t dt) + 9e^{2t}(2e^{2t})\,dt =$$

$3e^{3t} + 18e^{4t}$

$$\int_0^1 (3e^{3t} + 18e^{4t})\,dt =$$

$$= e^{3t} + 18\tfrac{1}{4}e^{4t}\Big|_0^1 = e^3 + \tfrac{18}{4}e^4 - \left(e^0 + \tfrac{18}{4}e^0\right) =$$

$$= e^3 + \tfrac{9}{2}e^4 - \left(1 + \tfrac{9}{2}\right).$$

Problem 10. Evaluate $\int_C \underset{=}{x}dx + \underset{=}{y}dy$, where C is the arc of the parabola $x = 4 - y^2$ from $(-5, -3)$ to $(3, 1)$.

$\vec{r}(t) = \langle \underbrace{4-t^2}_{x}, \underbrace{t}_{y} \rangle \qquad -3 \le t \le 1$

$dx = -2t\,dt \qquad dy = dt$

$$\int_{-3}^{1} (4-t^2)(-2t\,dt) + t\,dt = \int_{-3}^{1} (2t^3 - 7t)\,dt =$$

$-8t + 2t^3 + t$

$$= 2\frac{t^4}{4} - 7\frac{t^2}{2}\Big|_{-3}^{1} =$$

87
63
24

$$= \left[\frac{1}{2}1 - \frac{7}{2}1\right] - \left[\frac{1}{2}3^4 - \frac{7}{2}3^2\right] =$$

$$= -\frac{6}{2} - \frac{81}{2} + \frac{63}{2} = -\frac{24}{2} = -12$$

Problem 11. Evaluate $\int_C (x+y)dz + (y-x)dy + zdx$ where C is decribed by $x = t^4$, $y = t^3$, $z = t^2$, $0 \leq t \leq 1$.

$dx = 4t^3 dt$, $dy = 3t^2 dt$

$dz = 2t\,dt$

$$\int_0^1 (t^4 + t^3)(2t)dt + (t^3 - t^4)(3t^2)dt + t^2 4t^3 dt$$

$$2t^5 + 2t^4 + 3t^5 - 3t^6 + 4t^5$$

$$\int_0^1 (-3t^6 + 9t^5 + 2t^4)\,dt =$$

$$= -\frac{3}{7} + \frac{9}{6} + \frac{2}{5} = -\frac{3}{7} + \frac{3}{2} + \frac{2}{5} = \frac{-30+105+28}{70} = \frac{103}{70}$$

MATHEMATICS
TEXAS A&M UNIVERSITY

Instructor: Rosanna Pearlstein

Math Learning Center

Math 251 – Spring 2023
"Week-in-Review"

Section 16.3

In section 16.2, we learned how to find a line integral over a vector field $\mathbf{F}$ along a curve C that is parametrized by $\mathbf{r}(t)$, $a \le t \le b$.

Problem 1. Suppose we are moving a particle from the point $(0, 0)$ to the point $(2, 4)$ in a force field $\mathbf{F}(x, y) = \langle \overset{P}{y^2}, \overset{Q}{x} \rangle$. Find $\int_c \mathbf{F} \cdot d\mathbf{r}$,where $\quad Q_x - P_y = 1 - 2y$

a.) The particle travels along the line segment from $(0, 0)$ to $(2, 4)$.

b.) The particle travels along the curve $y = x^2$ from $(0, 0)$ to $(2, 4)$.

a) $\vec{r}(t) = \langle \overset{x}{2t}, \overset{y}{4t} \rangle \quad 0 \le t \le 1$

$\int_0^1 \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt = \ldots$

$\vec{F}(\vec{r}(t)) = \langle 16t^2, 2t \rangle$

$\vec{r}\,'(t) = \langle 2, 4 \rangle$

$\vec{F} \cdot \vec{r}\,' = 32t^2 + 8t$

$\int_0^1 (32t^2 + 8t)\, dt =$

$= \frac{32}{3} + \frac{8}{2} = \frac{32}{3} + 4 = \frac{44}{3}$

b) $\vec{r}(t) = \langle t, t^2 \rangle \quad 0 \le t \le 2$

$\int_0^2 \ldots$

$\vec{F}(\vec{r}(t)) = \langle t^4, t \rangle$

$\vec{r}\,'(t) = \langle 1, 2t \rangle$

$\vec{F} \cdot \vec{r}\,' = t^4 + 2t^2$

$\int_0^2 (t^4 + 2t^2)\, dt$

$\frac{t^5}{5} + 2\frac{t^3}{3} \Big|_0^2 = \frac{32}{5} + \frac{16}{3} = \frac{96 + 80}{15} = \frac{176}{15}$

different numbers.

Note: Although the end points are the same, the value of the line integral is **different** because the **paths** are different. In this section, we will learn under what conditions the line integral is independent of the path taken.

Definition: If $\mathbf{F}$ is a continuous vector field, we say that $\int_c \mathbf{F} \cdot d\mathbf{r}$ is **independent of path** if and only if $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ for any two paths C_1 and C_2 with the same starting and ending points. In other words, the line integral is the same **no matter what path** you travel on as long as the endpoints are the same.

Definition: A vector field $\mathbf{F}$ is called a conservative vector field if it is the gradient of some scalar funtion f, that is there exists a function f so that $\mathbf{F} = \nabla f$. We call f the **potential function**.

MATHEMATICS
TEXAS A&M UNIVERSITY

Math Learning Center

Instructor: Rosanna Pearlstein

Math 251 – Spring 2023
"Week-in-Review"

potential function

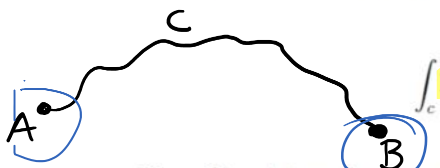
Problem 2. Consider $\boxed{f(x, y) = x^2y - y^3}$. Find the gradient and explain why it is conservative. What is the potential function?

by definition of conservative v. f.

Recall the Fundamental Theorem of Calculus tells us that $\int_a^b f'(x)dx = f(b) - f(a)$.

Since $\nabla f = \langle f_x, f_y \rangle$, we can think of the potential function, f, as some sort of antiderivative of ∇f. Hence $\int \mathbf{F} \cdot d\mathbf{r} = \int \nabla f \cdot d\mathbf{r}$.

Fundamental Theorem for Line Integrals: Let C be a smooth curve given by the vector function $\mathbf{r}(t)$, $a \le t \le b$. Let $\mathbf{F}$ be a conservative vector field. Let f be a differentiable function of two or three variables whose gradient vector, ∇f, is continuous on C. Then



$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \vec{\nabla f} \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$

Note: **Line integrals of conservative vectors fields are independent of path** because in a conservative vector field, the line integral is computed by only using the **endpoints** of the domain! Therefore, if we are in a conservative vector field, the line integral along a curve C in that vector field will be the same **no matter what curve we travel across** that connects the endpoints together. **WHICH MEANS WE DON'T EVEN NEED TO PARAMETERIZE THE CURVE!**

Question: How do we determine if a vector field is conservative, and if so, how do we find the potential function? The 'test for conservative' we use depends on whether $\mathbf{F}$ is in $\mathbb{R}^2$ or $\mathbb{R}^3$.

do curl $\vec{F}$, if it is $\vec{0}$ then $\vec{F}$ conserv.
and vice versa

Instructor: Rosanna Pearlstein

Math Learning Center

Math 251 – Spring 2023
"Week-in-Review"

Theorem: $\mathbf{F}(x,y) = \langle P(x,y), Q(x,y)\rangle = P\mathbf{i} + Q\mathbf{j}$ is a conservative vector field, where P and Q have continuous first-order partial derivatives on a domain D, if and only if $\dfrac{\partial Q}{\partial x} = \dfrac{\partial P}{\partial y}$.

Note: This above criteria to determine if a vector field is conservative works only for $\mathbb{R}^2$.

Problem 3. (a) Is $\mathbf{F}(x,y) = \langle \underbrace{3x^2 - 4y}_{P}, \underbrace{4y^2 - 2x}_{Q}\rangle$ a conservative vector field? (b) If so, find a function f so that $\mathbf{F} = \nabla f$.

(a) $\text{curl}\,\vec{F} = \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2-4y & 4y^2-2x & 0 \end{vmatrix} = (0-0)\hat{i} - (0-0)\hat{j} + (-2 - -4)\hat{k} =$

$= 2\hat{k} \neq 0 \Rightarrow$ not cons.

$Q_x = -2 \neq P_y = -4$

(b) n/a.

MATHEMATICS
TEXAS A&M UNIVERSITY

Math Learning Center

Instructor: Rosanna Pearlstein

Math 251 – Spring 2023
"Week-in-Review"

Problem 4. Is $\mathbf{F}(x,y) = \langle \underbrace{x+y}_{P}, \underbrace{x-2}_{Q} \rangle$ a conservative vector field? (a) If so, find a function f so that $\mathbf{F} = \nabla f$. (b)

(a) $Q_x = 1$, $P_y = 1$ so $\vec{F}$ is <u>conservative</u>.

(b) $\vec{F} = \vec{\nabla} f$

$\langle x+y, x-2 \rangle = \langle f_x, f_y \rangle$

$$\frac{\partial f}{\partial x} = x + y \Rightarrow f = \frac{x^2}{2} + xy + g(y)$$

$$\frac{\partial f}{\partial y} = x - 2 \Rightarrow f = xy - 2y + h(x)$$

Potential function

$$\boxed{f(x,y) = \frac{x^2}{2} + xy - 2y + C}$$

Always check!

P Q

Problem 5. Given $\mathbf{F}(x,y) = \langle 2xy^3, 3x^2y^2\rangle$. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is the curve given by $\mathbf{r}(t) = \langle t^3+2t^2-t, 3t^4-t^2\rangle$, $\boxed{0} \le t \le \boxed{2}$

$Q_x - P_y = 6xy^2 - 6xy^2 = 0 \Rightarrow$ cons.

Find potential function

$\langle 2xy^3, 3x^2y^2\rangle = \langle f_x, f_y\rangle$

integrate

$f = x^2y^3 + g(y)$

$f = x^2y^3 + h(x)$

$f(x,y) = x^2y^3 + C \rightarrow$ Pot. fnctn

CHECK! ✓

$\vec{r}(0) = \langle 0, 0\rangle$

$\vec{r}(2) = \langle 8+8-2, 3*16-4\rangle = \langle 14, 44\rangle$

$$\int_C \vec{F}\cdot d\vec{r} = f(14,44) - f(0,0) = 14^2 * 44^3 - 0$$

Problem 6. Let $\mathbf{F}(x,y) = \langle \overset{P}{3 + 2xy^2}, \overset{Q}{2x^2y} \rangle$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the arc of the hyperbola $y = \dfrac{1}{x}$ from $(1,1)$ to $\left(4, \dfrac{1}{4}\right)$.

$\vec{r}(t) = \langle t, \frac{1}{t} \rangle \quad 1 \le t \le 4$

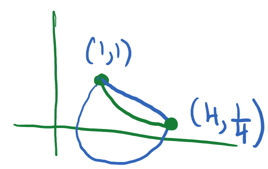
$\vec{F}(r(t)) = \langle 3 + 2t\frac{1}{t^2}, 2t^2\frac{1}{t} \rangle$

$\vec{r}\,'(t) = \langle 1, -\frac{1}{t^2} \rangle$

$Q_x - P_y = 4xy - 4xy = 0 \Rightarrow$ conservative

Pot. fnctn:

$\langle 3 + 2xy^2, 2x^2y \rangle = \langle f_x, f_y \rangle$



$f = 3x + x^2y^2 + g(y)$

$f = x^2y^2 + h(x)$

$f(x,y) = x^2y^2 + 3x + C$

$$\int_C \vec{F} \cdot d\vec{r} = f\left(4, \tfrac{1}{4}\right) - f(1,1) = \left(4^2\tfrac{1}{4^2} + 12\right) - (1+3) = 9$$

Instructor: Rosanna Pearlstein

Math Learning Center

Math 251 – Spring 2023
"Week-in-Review"

Problem 7. Given $\mathbf{F}(x,y) = \langle \overbrace{3x^2 - 4y}^{P}, \overbrace{4y^2 - 2x}^{Q} \rangle$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve given by $\underbrace{\mathbf{r}(t) = \langle t^2, t^2 + t - 2 \rangle}$, $0 \le t \le 1$.

$Q_x - P_y = -2 - (-4) \neq 0 \Rightarrow$ not cons.

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

$\vec{F}(\vec{r}(t)) = \langle 3t^4 - 4t^2 - 4t + 8, 4(t^2+t-2)^2 - 2t^2 \rangle$

$\vec{r}\,'(t) = \langle 2t, 2t+1 \rangle$

dot: $\vec{F} \cdot \vec{r}\,'$

MATHEMATICS
TEXAS A&M UNIVERSITY

Instructor: Rosanna Pearlstein

Math Learning Center

Math 251 – Spring 2023
"Week-in-Review"