MATHEMATICS
TEXAS A&M UNIVERSITY

Math Learning Center

Instructor: Rosanna Pearlstein

Math 251 – Spring 2023
"Week-in-Review"

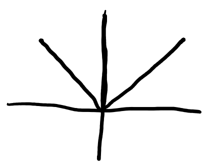
Wir 6: Sections 15.1, 15.2, 15.3

Section 15.1

Problem 1. Find $\int_0^{\pi/4} x\sin(3y)\,dy = x\left[-\frac{1}{3}\cos(3y)\right]_{y=0}^{\pi/4} =$

$$= -\frac{1}{3}x\left[\cos\frac{3\pi}{4} - \cos 0\right] = -\frac{x}{3}\left(-\frac{\sqrt{2}}{2} - 1\right) =$$

$$= \frac{x}{3}\left(1 + \frac{\sqrt{2}}{2}\right).$$



Problem 2. Find $\int_1^e \frac{y \ln(x)}{\boxed{x}} \boxed{dx}$

$u = \ln x$

$du = \frac{1}{x} dx$

$$y \int_{\ln 1}^{\ln e} u \, du = y \left. \frac{u^2}{2} \right|_0^1 = y \left(\frac{1}{2} - 0 \right) = \frac{1}{2} y.$$

Problem 3. Evaluate $\int_0^2 \left\{ \int_0^3 (xy + x + y)\, dy \right\} dx$ and $\int_0^3 \left\{ \int_0^2 (xy + x + y)\, dx \right\} dy$

$x\frac{1}{2}y^2 + xy + \frac{1}{2}y^2 \Big|_{y=0}^{3}$

$\underbrace{\frac{9}{2}x + 3x}+\frac{9}{2} - (0)$

$\int_0^2 \frac{15}{2}x + \frac{9}{2}\, dx$

$\frac{15}{2}\frac{x^2}{2} + \frac{9}{2}x \Big|_{x=0}^{2} =$

$= \frac{15}{\cancel{4}}\cancel{4} + \frac{9}{\cancel{2}}\cancel{2} - 0 =$

$= \boxed{24}$

$\frac{x^2}{2}y + \frac{x^2}{2} + yx \Big|_{x=0}^{2} =$

$= \frac{4}{2}y + \frac{4}{2} + 2y - 0$

$= 2y + 2 + 2y =$

$= \int_0^3 2 + 4y \quad dy$

$= 2y + 2y^2 \Big|_{y=0}^{3} =$

$= 6 + 18 - 0 =$

$= \boxed{24}$

Fubini's Theorem: If f is continuous on the rectangle $R = [a, b] \times [c, d]$, then

(1) $\iint_R f(x, y)\, dA = \int_a^b \int_c^d f(x, y)\, dydx = \int_c^d \int_a^b f(x, y)\, dxdy.$

(2) In the case where $f(x, y) = g(x)h(y)$, then

$$\iint_R f(x, y)\, dA = \int_a^b \int_c^d g(x)h(y)\, dydx = \left[\int_a^b g(x)\, dx\right]\left[\int_c^d h(y)\, dy\right]$$

Problem 4. Find $\iint_R \frac{x}{y^2}\, dA$, where $R = \overset{x}{[0, 4]} \times \overset{y}{[1, 2]}$ =

$$= \left\{\int_0^4 x\, dx\right\}\left\{\int_1^2 \frac{1}{y^2}\, dy\right\} =$$

$$= \left.\frac{x^2}{2}\right|_0^4 \left[-\frac{1}{y}\right]_1^2 = \left(\overset{8}{\cancel{\tfrac{16}{2}}} - 0\right)\left(-\frac{1}{2} - -\frac{1}{1}\right)$$

$$8 \cdot \frac{1}{2} = 4$$

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Problem 5. Find $\iint_R (x \sec^2 y)\, dA$, where $R = \{(x,y) | 0 \le x \le 2, 1 \le y \le \frac{\pi}{4}\}$

$$\left(\int_0^2 x\,dx\right)\left(\int_1^{\pi/4} \sec^2 y\,dy\right) = \frac{x^2}{2}\Big|_0^2 \quad \tan y\Big|_1^{\pi/4} =$$

$$= \left(\frac{4}{2} - 0\right)\left(\overset{\tan\frac{\pi}{4}}{1} - \tan 1\right) = 2\left(1 - \tan 1\right)$$

Problem 6. Find $\iint_R e^{2x+y}\,dA$, where $R = [0, \ln 2] \times [0, \ln 3]$

$$e^{2x} \cdot e^{y}$$

$$\int_0^{\ln 2} e^{2x}\,dx \cdot \int_0^{\ln 3} e^{y}\,dy = \frac{1}{2} e^{2x}\Big|_{x=0}^{\ln 2} \cdot e^{y}\Big|_{y=0}^{\ln 3} =$$

$$= \frac{1}{2}\left(e^{2\ln 2} - e^{0}\right)\left(e^{\ln 3} - e^{0}\right) = \frac{1}{2}(4-1)(3-1) =$$

$$= \frac{1}{\cancel{2}}\, 3 \cdot \cancel{2} = 3$$

$$\left(e^{\ln 2}\right)^2$$

$$e^{(\ln 2^2)}$$

Problem 7. Find $\iint_R (y\cos(xy))\,dA$, where $R = [0,2] \times [0,\pi]$

$$\underbrace{\int_0^2 \int_0^\pi y \cos xy \, dy\, dx}_{\text{Integr. By Parts}} = \int_0^\pi \left\{ \int_0^2 y \cos xy \, dx \right\} dy$$

$$\cancel{y}\frac{1}{\cancel{y}} \sin xy \Big|_{x=0}^{2} =$$

$$= \sin 2y - \sin 0 = \sin 2y$$

$$\int_0^\pi \sin 2y \, dy = -\frac{1}{2}\cos 2y \Big|_0^\pi =$$

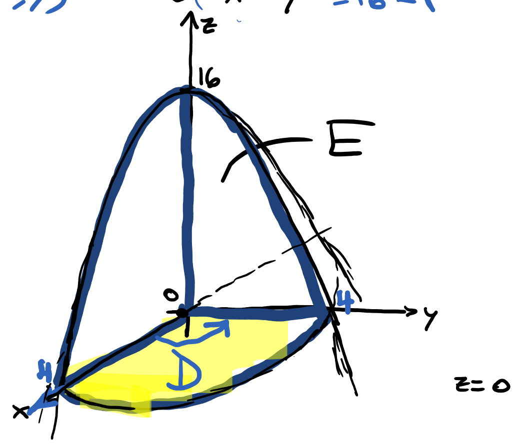
$$= -\frac{1}{2}(1-1) = 0$$

$f = ?$ $D = ?$

Problem 8. Find the volume of the solid S that is bounded by the paraboloid $x^2 + y^2 + z = 16$, $z = 0$, $0 \le x \le 4$, $0 \le y \le 4$.

$f(x,y) = z = 16 - x^2 - y^2 = 16 - r^2$

$$\text{Vol } E = \iint_D f(x,y)\, dA$$



$$D: \begin{cases} 0 \le r \le 4 \\ 0 \le \theta \le \frac{\pi}{2} \end{cases}$$

$$\int_0^{\pi/2} \left\{ \int_0^4 (16 - r^2)\, r\, dr \right\} d\theta$$

$$\int (16r - r^3)\, dr$$

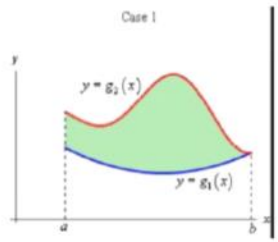
$$16\frac{r^2}{2} - \frac{r^4}{4} \Big|_{r=0}^{4} = 16 \cdot \frac{16}{2} - \frac{\cancel{4} \cdot 4^3}{\cancel{4}} - 0$$

$$128 - 64 = 64$$

$$64 \cdot \frac{\pi}{2} = \boxed{32\pi}$$

Section 15.2

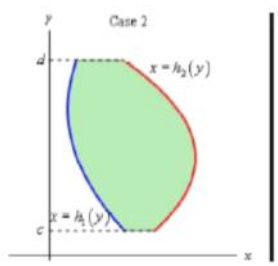
Type I: A plane region D is said to be of type I if it lies between the graphs of two continuous functions of x, that is $D = \{(x, y) | a \leq x \leq b, g_1(x) \leq y \leq g_2(x)\}$.



If f is continuous on a type I region $D = \{(x, y) | a \leq x \leq b, g_1(x) \leq y \leq g_2(x)\}$, then

$$\iint_R f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dydx$$

Type II: A plane region D is said to be of type II if it lies between the graphs of two continuous functions of y, that is $D = \{(x, y) | c \leq y \leq d, h_1(y) \leq x \leq h_2(y)\}$.

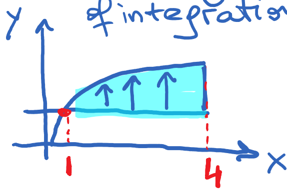


If f is continuous on a type II region $D = \{(x, y) | c \leq y \leq d, h_1(y) \leq x \leq h_2(y)\}$, then

$$\iint_D f(x, y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\, dxdy$$

Problem 9. Evaluate $\int_1^4 \int_1^{\sqrt{x}} (x+y)\, dy dx$

EXTRA: sketch region of integration



$$xy + \frac{1}{2}y^2 \Big|_{y=1}^{y=\sqrt{x}} = x\sqrt{x} + \frac{1}{2}x - \left(x + \frac{1}{2}\right)$$

$$= \int_1^4 x^{3/2} - \frac{1}{2}x - \frac{1}{2}\, dx = \frac{2}{5}x^{5/2} - \frac{1}{2}\frac{x^2}{2} - \frac{1}{2}x \Big|_{x=1}^{4} =$$

$$= \frac{2}{5}4^{5/2} - \frac{16}{4} - \frac{4}{2} - \left(\frac{2}{5} - \frac{1}{4} - \frac{1}{2}\right)$$

$2^5 = 32$

$$= \frac{64}{5} - 4 - 2 - \frac{2}{5} + \frac{1}{4} + \frac{1}{2}$$

$$\frac{-8+5+10}{20}$$

$$= \frac{62}{5} - 6 + \frac{7}{20} =$$

$$= \frac{248-120+7}{20} = \frac{143}{20}.$$

Extra: Sketch

Type II

Problem 10. Evaluate $\int_0^1 \int_0^y (3 + x^2 y)\, dx\, dy$

$$3x + \frac{x^3}{3} y \Big|_{x=0}^{y} =$$

$$= \int_0^1 3y + \frac{1}{3} y^4 - 0 \quad dy$$

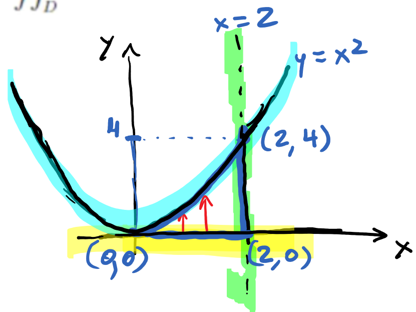
$$3 \frac{1}{2} y^2 + \frac{1}{3} \frac{1}{5} y^5 \Big|_{y=0}^{1} = \frac{3}{2} + \frac{1}{15} = \frac{45+2}{30} = \frac{47}{30}.$$

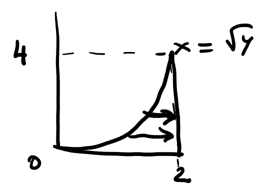
Extra: solve as "dy dx" (type I)

Problem 11. Sketch the region of integration and evaluate $\iint_D xe^y \, dA$ where D is the region bounded by $y = 0$, $y = x^2$ and $x = 2$

Type I $\begin{cases} 0 \le x \le 2 \\ 0 \le y \le x^2 \end{cases}$

Type II $\begin{cases} 0 \le y \le 4 \\ \sqrt{y} \le x \le 2 \end{cases}$





$$\int_0^2 \int_0^{x^2} xe^y \, dy \, dx$$

$$xe^y \Big|_{y=0}^{x^2} = xe^{x^2} - xe^0$$

$$\int_0^2 (xe^{x^2} - x) \, dx$$

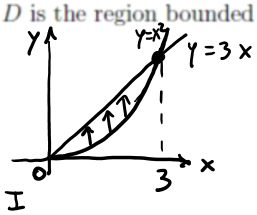
$u = x^2$
$du = 2(x\,dx)$

$$\int_{0^2}^{2^2} \frac{1}{2} e^u \, du - \frac{x^2}{2}\Big|_0^2 =$$

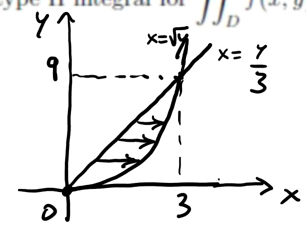
$$= \frac{1}{2} e^u \Big|_0^4 - \left(\frac{4}{2} - \frac{0}{2}\right) = \frac{1}{2}\left(e^4 - e^0\right) - 2 =$$

$$= \frac{1}{2} e^4 - \frac{5}{2}.$$

Problem 12. Set up but do not evaluate both a type I and type II integral for $\iint_D f(x,y)\,dA$, where D is the region bounded by $y = x^2$ and $y = 3x$.



$x^2 = 3x$
$x = 0, 3$



Type I

$$\begin{cases} 0 \le x \le 3 \\ x^2 \le y \le 3x \end{cases}$$

Type 2

$$\begin{cases} 0 \le y \le 9 \\ \frac{y}{3} \le x \le \sqrt{y} \end{cases}$$

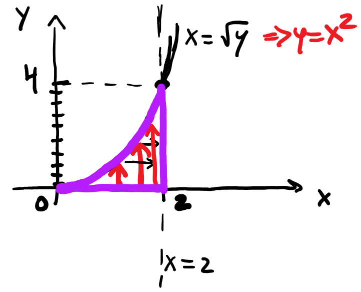
$$\int_0^3 \int_{x^2}^{3x} f(x,y)\,dy\,dx = \int_0^9 \int_{y/3}^{\sqrt{y}} f(x,y)\,dx\,dy$$

Problem 13. Sketch the region of integration and change the order of integration.

(i) $\displaystyle\int_0^4 \int_{\sqrt{y}}^2 f(x, y)\, dx dy$

(ii) $\displaystyle\int_1^2 \int_0^{\ln x} f(x, y)\, dy dx$

(i.)

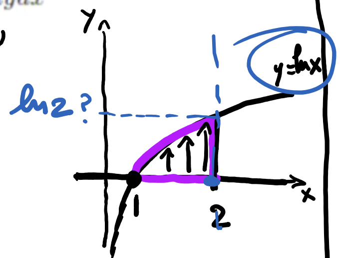


$$\begin{cases} 0 \le x \le 2 \\ 0 \le y \le x^2 \end{cases}$$

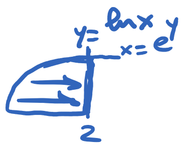
$$\int_0^2 \int_0^{x^2} f(x,y)\, dy\, dx \,.$$

(ii.)

$$\begin{cases} 0 \le y \le \ln 2 \\ e^y \le x \le 2 \end{cases}$$

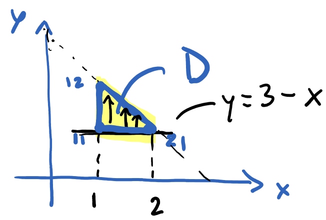


$$\int_0^{\ln 2} \int_{e^y}^{2} f(x,y)\, dx\, dy$$



Problem 14. Set up but do not evaluate a double itegral that gives the volume of the solid under the surface $z = xy$ and above the triangle with vertices $(1,1)$, $(1,2)$ and $(2,1)$

$$\text{Vol} = \iint_D f(x,y)\, dA \qquad D = ? \quad f = ? \qquad f(x,y) = xy$$



$$\text{Type I} \begin{cases} 1 \le x \le 2 \\ 1 \le y \le 3-x \end{cases}$$

Extra: do as a type 2

$$\int_1^2 \int_1^{3-x} xy\, dy\, dx =$$

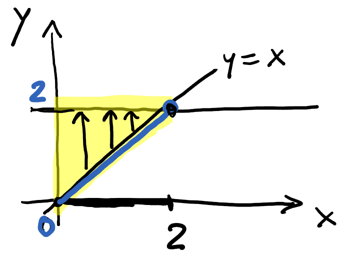
$$\frac{1}{2} x y^2 \Big|_{y=1}^{3-x} = \frac{1}{2} x \left[(3-x)^2 - 1^2 \right] =$$

$$= \frac{1}{2} x \left[9 - 6x + x^2 - 1 \right] = \frac{1}{2} x \left(8 - 6x + x^2 \right)$$

$$\int_1^2 4x - 3x^2 + \frac{1}{2} x^3\, dx = 2x^2 - x^3 + \frac{1}{2} \frac{1}{4} x^4 \Big|_{x=1}^{2} =$$

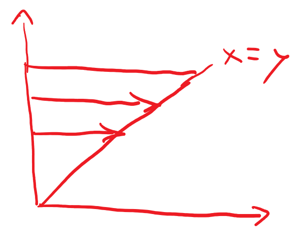
$$= \cancel{8} - \cancel{8} + \frac{1}{\cancel{8}} \cancel{16}^{\,2} - \left(2 - 1 + \frac{1}{8} \right) = \cancel{2} - \cancel{2} + 1 - \frac{1}{8} = \frac{7}{8}$$

Problem 15. Evaluate $\int_0^2 \int_x^2 e^{-y^2}\, dy dx$ must switch to $dx\,dy$



$$\begin{cases} 0 \le y \le 2 \\ 0 \le x \le y \end{cases}$$

$$\int_0^2 \left\{ \int_0^y e^{-y^2}\, dx \right\} dy$$



$$x e^{-y^2} \Big|_{x=0}^{y} = y e^{-y^2} - 0 e^{-y^2}$$

$$\int_0^2 y e^{-y^2}\, dy \qquad u = -y^2$$

$$du = -2(y\, dy)$$

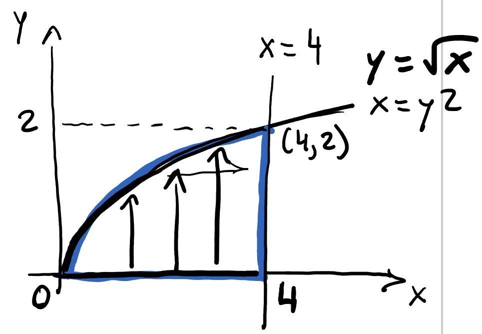
$$\int_{-0^2}^{-(2^2)} -\frac{1}{2} e^u\, du = -\frac{1}{2}\int_0^{-4} e^u\, du = \frac{1}{2}\int_{-4}^{0} e^u\, du =$$

$$= \frac{1}{2}\left(e^0 - e^{-4}\right) = \frac{1}{2}\left(1 - \frac{1}{e^4}\right).$$

Problem 16. Evaluate $\int_0^2 \int_{y^2}^4 \sqrt{x}\sin x \, d\cancel{y}\, x \, d\cancel{x}\, y$

TYPO, IT SHOULD SAY DXDY

$$\begin{cases} 0 \le x \le 4 \\ 0 \le y \le \sqrt{x} \end{cases}$$



$$\int_0^4 \int_0^{\sqrt{x}} \sqrt{x}\sin x \, dy\, dx$$

$$y(\sqrt{x}\sin x)\Big|_{y=0}^{\sqrt{x}} = x\sin x - 0$$

deriv	integr.
x	$\sin x$
1 $-$	$-\cos x$
0	$-\sin x$

$$\int_0^4 x\sin x\, dx =$$

$$-x\cos x + \sin x \Big|_{x=0}^{4} =$$

$$= -4\cos 4 + \sin 4 + 0 - 0$$

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Section 15.3

Recall: If $P(x, y)$ is a point in the xy-plane, we can represent the point P in polar form: Let r be the distance from O to P and let θ be the angle between the polar axis and the line OP. Then the point P is represented by the ordered pair (r, θ), and r, θ are called the **polar coordinates** of P.

Connecting polar coordinates with rectangular coordinates:

→ a.) $x = r\cos(\theta)$, $y = r\sin(\theta)$

b.) $\tan(\theta) = \frac{y}{x}$, thus $\boxed{\theta = \arctan\left(\frac{y}{x}\right)}$.

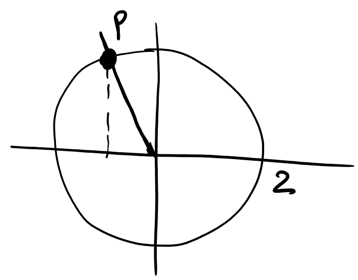
c.) $x^2 + y^2 = r^2$

Problem 1. Find the cartesian coordinates of the polar point $\left(\overset{r}{2}, \overset{\theta}{\frac{2\pi}{3}}\right)$. P

$x = 2\cos\frac{2\pi}{3} = 2\left(-\frac{1}{2}\right) = -1$

$y = 2\sin\frac{2\pi}{3} = 2\frac{\sqrt{3}}{2} = \sqrt{3}$

$P(-1, \sqrt{3})$



Problem 2. Find the polar coordinates of the rectangular point $(\overset{x}{\sqrt{3}}, \overset{y}{-1})$.

$r^2 = \sqrt{3}^2 + (-1)^2 = 3+1 = 4 \qquad r=2$

$\theta = \arctan \frac{-1}{\sqrt{3}} = -\arctan \frac{1}{\sqrt{3}} = -\frac{\pi}{6}.$

$(2, -\frac{\pi}{6})$ & many others...

Problem 3. Find a cartesian equation for the curve described by $r = 2\sin\theta$.

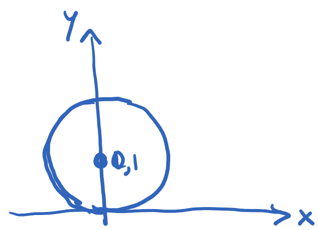
$$r = 2\sin\theta$$ Times "r"

$$r^2 = 2\underbrace{r\sin\theta}$$

$$x^2 + y^2 = 2y$$

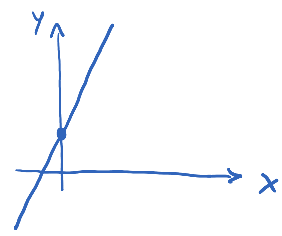
$$x^2 + y^2 - 2y + 1 = 0 + 1$$

$$x^2 + (y-1)^2 = 1$$



$\rightarrow r = r(\theta)$

Problem 4. Find a polar equation for $y = 1 + 3x$



$$r\sin\theta = 1 + 3r\cos\theta$$

$$r(\sin\theta - 3\cos\theta) = 1$$

$$r = \frac{1}{\sin\theta - 3\cos\theta}$$

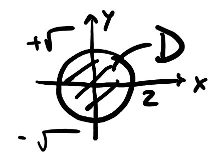
Change to Polar Coordinates in a Double Integral: If f is continuous on a polar rectangle R given by $0 \le a \le r \le b$, $\alpha \le \theta \le \beta$, where $0 \le \beta - \alpha \le 2\pi$, then

dxdy → rdrdθ

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

Problem 5. Evaluate $\iint_R (x+2)\,dA$, where R is the region bounded by the circle $x^2+y^2=4$.

$$D: \begin{cases} 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \end{cases}$$



$$\int_0^{2\pi}\int_0^2 (r\cos\theta + 2)\, r\,dr\,d\theta$$

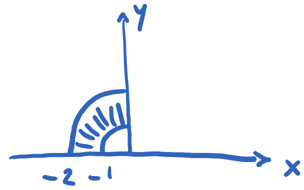
$$\int (r^2\cos\theta + 2r)\,dr$$

$$\frac{r^3}{3}\cos\theta + r^2 \Big|_{r=0}^{2} = \int_0^{2\pi} \frac{8}{3}\cos\theta + 4 - 0 \quad d\theta$$

$$\frac{8}{3}\sin\theta + 4\theta \Big|_{\theta=0}^{2\pi} = 4(2\pi - 0) = 8\pi.$$

Problem 6. Evaluate $\iint_R 4y\,dA$, where R is the region in the second quadrant bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

$$\begin{cases} \frac{\pi}{2} \le \theta \le \pi \\ 1 \le r \le 2 \end{cases}$$



$$\int_{\frac{\pi}{2}}^{\pi} \int_{1}^{2} 4r\sin\theta \; r\,dr\,d\theta$$

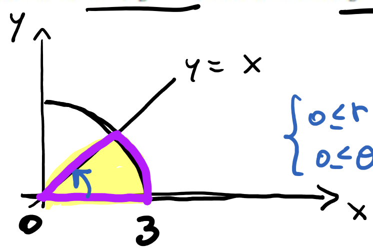
$$\frac{4}{3} r^3 \sin\theta \Big|_{r=1}^{2} =$$

$$= \frac{4}{3}\sin\theta\,(8-1) = \int_{\pi/2}^{\pi} \frac{28}{3}\sin\theta \; d\theta$$

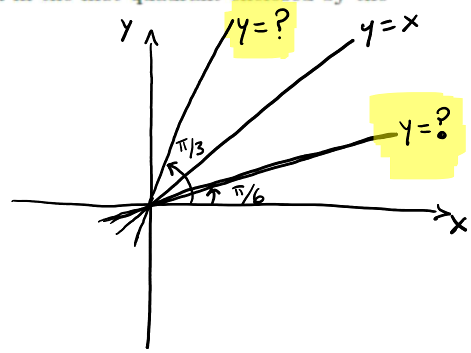
$$-\frac{28}{3}\cos\theta \Big|_{\theta=\frac{\pi}{2}}^{\pi} =$$

$$= -\frac{28}{3}(-1-0) = \frac{28}{3}.$$

Problem 7. Evaluate $\iint_R 3x^2\,dA$, where R is the region in the first quadrant enclosed by the by the circle $x^2+y^2=9$ and the lines $y=0$ and $y=x$.



$$\begin{cases} 0 \le r \le 3 \\ 0 \le \theta \le \frac{\pi}{4} \end{cases}$$



$$\int_0^{\pi/4} \left\{ \int_0^3 3r^2 \cos^2\theta \; r\,dr \right\} d\theta$$

$$\cos^2\theta \left(3\,\frac{r^4}{4}\right)\Bigg|_{r=0}^{3} =$$

$$= \cos^2\theta \left(\frac{3}{4}\right)\left(3^4 - 0\right) \qquad \frac{81}{\underline{3}}$$

$$= \int_0^{\pi/4} \frac{243}{4} \cos^2\theta \quad d\theta$$

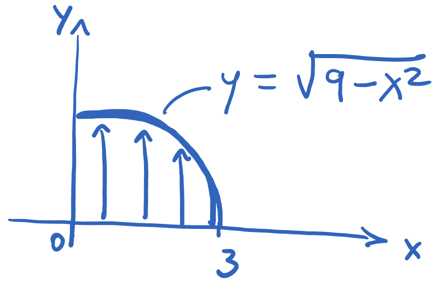
$$\frac{243}{4} \int_0^{\pi/4} \frac{1}{2}\left(1+\cos 2\theta\right) d\theta = \frac{243}{8}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/4} =$$

$$= \frac{243}{8}\left[\frac{\pi}{4} + \frac{1}{2}\overbrace{(1)}^{\sin\frac{\pi}{2}} - 0 - 0\right]$$

$$\frac{243}{8}\left(\frac{\pi}{4} + \frac{1}{2}\right)$$

Problem 8. Change $\int_0^3 \int_0^{\sqrt{9-x^2}} x^2\,dy\,dx$ to a polar double integral. Do not evaluate.

Change to polar



$$\begin{cases} 0 \le r \le 3 \\ 0 \le \theta \le \frac{\pi}{2} \end{cases}$$

$$\int_0^{\pi/2}\int_0^3 r^2\cos^2\theta \; r\,dr\,d\theta$$

$$\cos^2\theta \left.\frac{r^4}{4}\right|_{r=0}^{3} = \int_0^{\pi/2} \cos^2\theta\left(\frac{81}{4}-0\right)d\theta$$

$$\int \frac{81}{4}\,\frac{1}{2}\left(1+\cos 2\theta\right)d\theta$$

$$\frac{81}{8}\left[\theta + \underbrace{\frac{1}{2}\sin 2\theta}_{\text{zero}}\right]_0^{\pi/2} = \frac{81}{8}\left(\frac{\pi}{2}-0\right) = \frac{81}{16}\pi.$$

Problem 9. Change $\int_0^4 \int_0^{\sqrt{4x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$ to a polar double integral. Do not evaluate.

$\boxed{y = \sqrt{4x - x^2}}$

$y^2 - 4 = 4x - x^2 - 4$

$y^2 - 4 = -(x-2)^2$

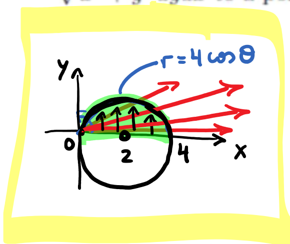
$(x-2)^2 + y^2 = 4$

$y^2 = 4x - x^2$

$x^2 + y^2 = 4x$

$r^2 = 4r\cos\theta$

$\boxed{r = 4\cos\theta}$



$$\begin{cases} 0 \le r \le 4\cos\theta \\ 0 \le \theta \le \frac{\pi}{2} \end{cases}$$

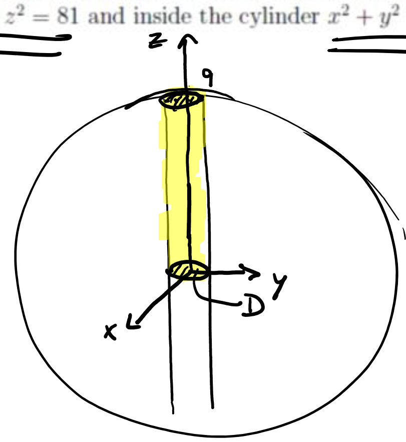
θ	$4\cos\theta$
0	4
$\frac{\pi}{6}$	---
$\frac{\pi}{4}$	---
$\frac{\pi}{3}$	---
$\frac{\pi}{2}$	0

$$\int_0^{\pi/2} \int_0^{4\cos\theta} \underbrace{\sqrt{r^2}\; r}_{r^2}\, dr\, d\theta$$

Problem 10. Set up but do not evaluate a double integral that gives the volume of the solid that lies above the xy-plane, below the sphere $x^2 + y^2 + z^2 = 81$ and inside the cylinder $x^2 + y^2 = 4$

$$Vol = \iint_D f(x,y)\, dA$$

$$D: \begin{cases} 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \end{cases}$$



$$f(x,y) = +\sqrt{81 - x^2 - y^2}$$

$$Vol = \int_0^{2\pi} \int_0^{2} \sqrt{81 - r^2}\; r\, dr\, d\theta$$

$z=20-r^2$

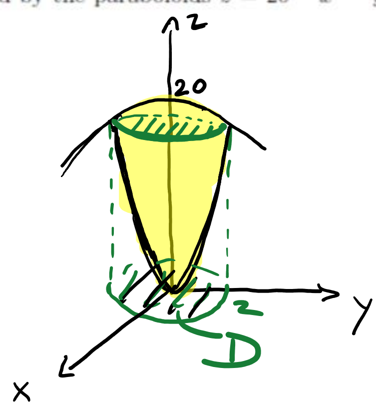
Problem 11. Find the volume of the solid bounded by the paraboloids $z = 20 - x^2 - y^2$ and $z = 4x^2 + 4y^2$.

$z = 4r^2$

$20 - r^2 = 4r^2$

$20 = 5r^2$

$4 = r^2 \Rightarrow r = 2$



$\text{Vol} = \iint_D f(x,y)\,dA$ Top – Bottom

$$V = \int_0^{2\pi}\int_0^2 \left[(20 - r^2) - (4r^2)\right] r\,dr\,d\theta$$

$$2\pi \int_0^2 (20r - 5r^3)\,dr$$

$$20\frac{r^2}{2} - 5\frac{r^4}{4}\Big|_0^2$$

$$20\frac{4}{2} - 5\frac{16}{4} - 0$$

$$40 - 20$$

$$2\pi(20)$$

40π