MATHEMATICS
TEXAS A&M UNIVERSITY

Instructor: Rosanna Pearlstein

Math Learning Center

Math 251 – Spring 2023
"Week-in-Review"

Wir 4: Sections 14.1, 14.3, 14.4

Section 14.1

Monday, February 20, 2023 5:51 P

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Problem 1. Find and sketch the domain of the following functions.

a.) $f(x, y) = \sqrt{4x - 2y}$

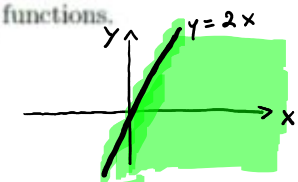
b.) $f(x, y) = \ln(y - 3x)$

c.) $f(x, y) = \sqrt[4]{xy}$

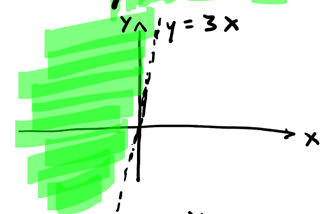
d.) $f(x, y) = \dfrac{\sqrt{8 - x^2 - y^2}}{x + 2y}$

e.) $f(x, y) = \dfrac{1}{\sqrt{x + 2y}} + \sqrt{4 - x^2 - y^2}$

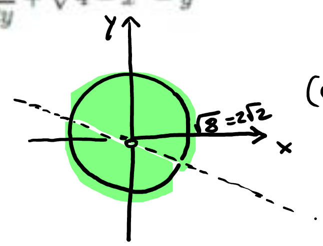
(a) $4x - 2y \geqslant 0$
$4x \geqslant 2y$
$2x \geqslant y$



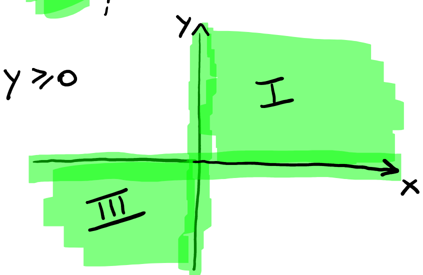
(b) $y - 3x > 0$
$y > 3x$



(d) $\begin{cases} 8 - x^2 - y^2 \geqslant 0 \\ x + 2y \neq 0 \end{cases}$
$x^2 + y^2 \leq 8$
$y \neq -\frac{x}{2}$

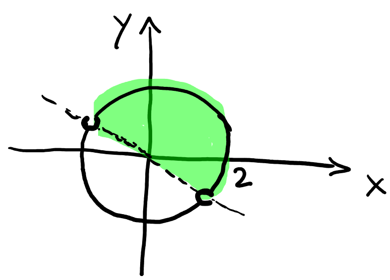


(c) $xy \geqslant 0$



(e) $\begin{cases} x + 2y > 0 \\ 4 - x^2 - y^2 \geqslant 0 \end{cases}$

$\begin{cases} y > -\frac{x}{2} \\ x^2 + y^2 \leq 4 \end{cases}$



Problem 2. Sketch the graph of the following surfaces:

a.) $2z = x + 3y - 6$

b.) $z = x^2 + 6y^2$

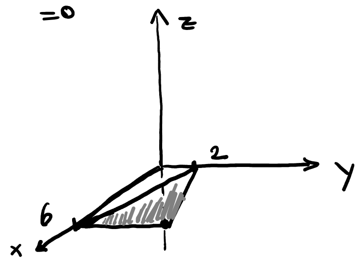
c.) $z = y^2$

d.) $z = 7 - x^2 - 2y^2$ $= 6$

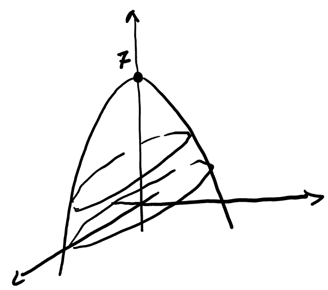
$x^2 + 2y^2 = 7 - 6 = 1$

(a) $-x - 3y + 2z = -6$

$\underbrace{x}_{=} + 3\underbrace{y}_{} - 2\underbrace{z}_{0} = \underbrace{6}_{=0}$

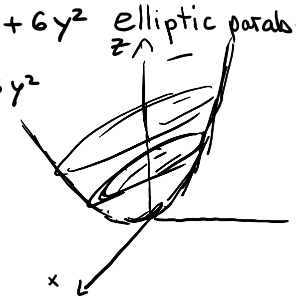


(d)



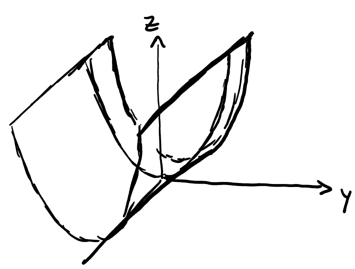
(b) $z = x^2 + 6y^2$ elliptic paraboloid

$z = 1 = x^2 + 6y^2$



(c) $z = y^2$

Parabolic cylinder



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Problem 3. Sketch several level curves for the following surfaces:

a.) $f(x, y) = 2 + 4x - y$ $= k$

b.) $f(x, y) = x + y^2$

c.) $f(x, y) = \sqrt{9 - x^2 - y^2}$

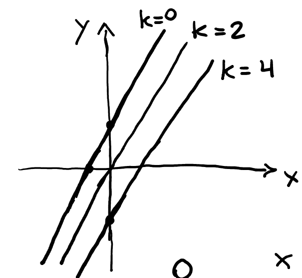
d.) $f(x, y) = 8\sqrt{x^2 - y^2}$

(a) $2 + 4x - y = 0, 2, 4$

$y = 4x + 2$

$y = 4x$

$y = -2 + 4x$



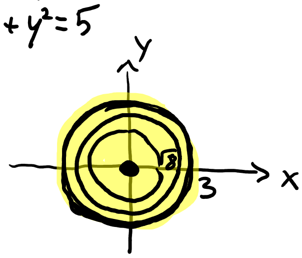
c) $\sqrt{9-x^2-y^2}^{\,2} = 0^2, 1^2, 2^2$

$9 - x^2 - y^2 = 0, 1, 4$

$x^2+y^2=9$

$x^2+y^2=8$

$x^2+y^2=5$

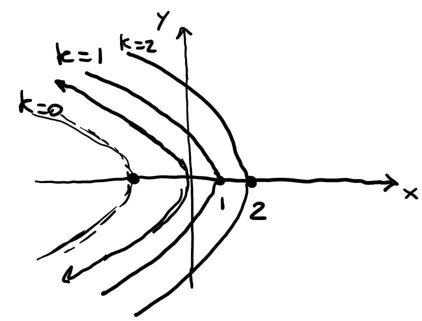


(b) $x + y^2 = 0, 1, 2$

$x = -y^2$

$x = 1 - y^2$

$x = 2 - y^2$



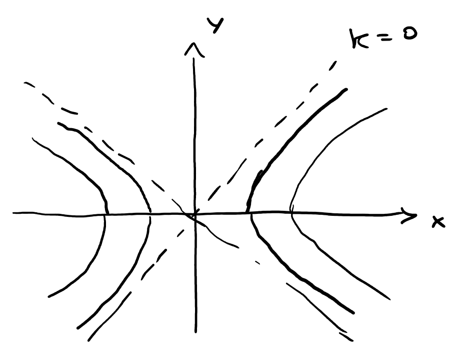
d)

$8\sqrt{x^2-y^2} = 0, 8, 32$

$\sqrt{x^2-y^2}^{\,2} = 0^2, 1^2, 4^2$

$x^2 - y^2 = 0, 1, 16$

$x = \pm y$



$x^2 = 1 + y^2$

$x = \pm\sqrt{1 + y^2}$

$x = \pm \sqrt{16+y^2}$ $\qquad$ $x^2 \geq y^2$

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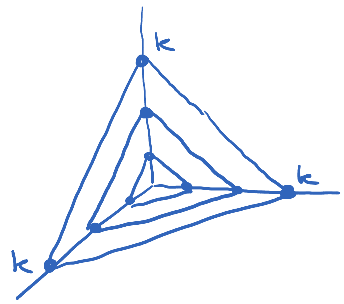
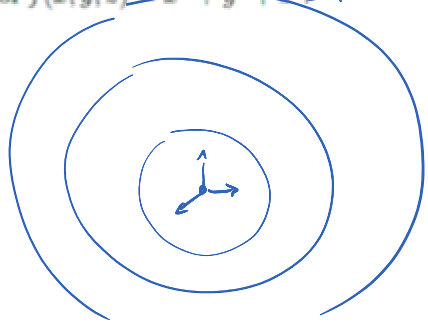
Problem 4. Describe the level surfaces of $f(x, y, z) = x + y + z. = k$

planes with all intercepts equal to k.



Problem 5. Describe the level surfaces of $f(x, y, z) = x^2 + y^2 + z^2 = k$

Concentric spheres
with center the origin.



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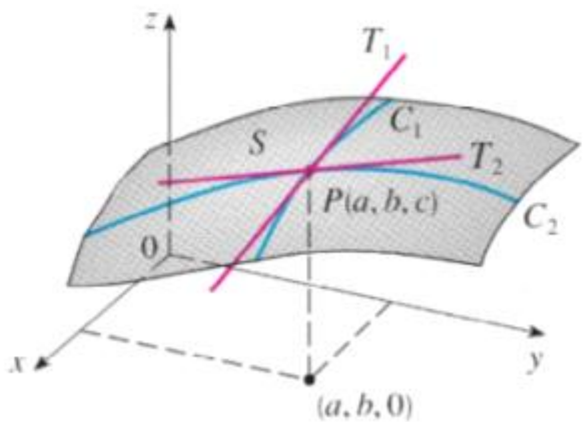
Section 14.3



Problem 6. Find $f_x(-1, 2)$ and $f_y(-1, 2)$ for $f(x, y) = x^3 - y^4 - 6x^2y^3$

$$\frac{\partial f}{\partial x} = 3x^2 - 0 - 12xy^3 \quad \longrightarrow \quad 3 + 12 \cdot 8 = 3 + 96 = 99 = f_x(-1,2)$$

at $(-1, 2)$

$$\frac{\partial f}{\partial y} = 0 - 4y^3 - 18x^2y^2 \quad \longrightarrow \quad -4(8) - 18(4) = 32 - 72 = -40 = f_y(-1,2)$$

Problem 7. Find $f_x(x,y)$ and $f_y(x,y)$ for $f(x,y) = x^2 e^{\cos(2x^4y^2)}$

$$f_x(x,y) = 2x\, e^{\cos(2x^4y^2)} + \left[x^2\, e^{\cos(2x^4y^2)}\right]\left(-\sin(2x^4y^2)\right)\left(8x^3y^2\right)$$

$$f_y(x,y) = x^2\, e^{\cos(2x^4y^2)}\left(-\sin(2x^4y^2)\right)\left(4x^4y\right)$$

Problem 8. If $f(x, y) = ye^{-x} + 2x$, find $\left.\dfrac{\partial f}{\partial x}\right|_{(1,0)}$ and $\left.\dfrac{\partial f}{\partial y}\right|_{(1,0)}$

$$\frac{\partial f}{\partial x} = -ye^{-x} + 2 \qquad \left.\frac{\partial f}{\partial x}\right|_{(1,0)} = 0 + 2 = 2$$

$$\frac{\partial f}{\partial y} = e^{-x} + 0 \qquad \left.\frac{\partial f}{\partial y}\right|_{(1,0)} = e^{-1} = \frac{1}{e}$$

$\frac{d}{dt}\left(\frac{1}{t}\right) = -\frac{1}{t^2}$

Problem 9. Find all higher order partial derivatives for $f(x, y) = \ln(2x + 3y)$

up to order 2

f

$$f_x = \frac{2}{2x+3y}$$

$$f_{xx} = \frac{-4}{(2x+3y)^2}$$

$$f_{xy} = \frac{-6}{(2x+3y)^2}$$

$$f_y = \frac{3}{2x+3y}$$

$$f_{yx} = \frac{-6}{(2x+3y)^2}$$

$$f_{yy} = \frac{-9}{(2x+3y)^2}$$

Clairaut's Theorem.

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Section 14.4

$z = f(x,y)$

Problem 10. Find the differential of $z = x^2 + 2y^2 + 4xy$ at the point $(1, 2)$.

$$df = f_x\,dx + f_y\,dy$$

$$f_x = 2x + 0 + 4y$$

$$f_y = 0 + 4y + 4x$$

$$df = (2x + 4y)dx + (4y + 4x)dy$$

$$2 + 8$$

$$8 + 4$$

$$df(1,2) = 10\,dx + 12\,dy$$

Problem 11. Find the differential of $f(x, y, z) = x^2y^3z^4$.

$$df = f_x\,dx + f_y\,dy + f_z\,dz =$$

$$= 2xy^3z^4\,dx + 3x^2y^2z^4\,dy + 4x^2y^3z^3\,dz.$$

$z_0 = (-1)^3 - 3(1^2) = -1 - 3 = -4$

$x_0 \quad y_0$

Problem 12. Find an equation of the tangent plane to the surface $z = x^3 - 3y^2$ at point $(-1, 1)$.

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$\boxed{z + 4 = 3(x+1) - 6(y-1)}$$

$f_x = 3x^2$, $f_x(-1,1) = 3$

$f_y = -6y$, $f_y(-1,1) = -6$

$f(2,2) = e^{2-2} = e^0 = 1$

Problem 13. Find an equation of the tangent plane to the surface $z = e^{x-y}$ at point $(2, 2, 1)$. What is the equation of the normal line to this tangent plane at point $(2, 2, 1)$?

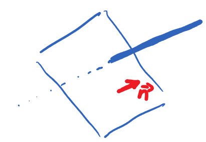
$z_0 = 1$

$f_x = e^{x-y}$

$f_x(2,2) = e^0 = 1$

$f_y = -e^{x-y}$

$f_y(2,2) = -e^0 = -1$



$$z - 1 = 1(x-2) - 1(y-2)$$

$$z - 1 = x - \cancel{2} - y + \cancel{2}$$

$$-x + y + z - 1 = 0 \qquad \vec{n} = \langle -1, 1, 1 \rangle = \vec{v} \text{ of the line.}$$

$$\begin{cases} x = 2 - t \\ y = 2 + t \\ z = 1 + t \end{cases} \quad \text{normal line.}$$

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Problem 14. Using the tangent plane to the graph of $f(x,y) = \sqrt{24 - x^2 - y^2}$ at point $(2,2)$, approximate $f(2.09, 1.93)$.

$x_0 \; y_0$

$$L(x,y) = z_0 + f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0)$$

same as the tangent plane

$$z_0 = f(2,2) = \sqrt{24-4-4} = \sqrt{16} = 4$$

$$f_x = \frac{-\cancel{2}x}{\cancel{2}\sqrt{24-x^2-y^2}} \Big\} \frac{-2}{4} = -\frac{1}{2} \qquad f_y = \frac{-y}{\sqrt{24-x^2-y^2}} \Big\} -\frac{1}{2}$$

$$L(x,y) = 4 - \frac{1}{2}(\overset{2.09}{x} - 2) - \frac{1}{2}(\overset{1.93}{y} - 2)$$

$$\underbrace{\sqrt{24 - (2.09^2) - (1.93^2)}}_{3.988} \approx 4 - \frac{1}{2}\frac{9}{100} - \frac{1}{2}\left(-\frac{7}{100}\right)$$

$$4 - \frac{9}{200} + \frac{7}{200} = \frac{800-9+7}{200} = \frac{798}{200} \approx 3.99$$

$(2^3 - 2(1^4) + 4(1^5))^3 =$

$= (8-2+4)^3 = 10^3 = 1{,}000$

Problem 15. Use differentials to approximate $((1.97)^3 - 2(0.9)^4 + 4(1.01)^5)^3$.

$f(x,y,z) = (x^3 - 2y^4 + 4z^5)^3$

$x_0 = 2$
$y_0 = 1$
$z_0 = 1$

$df = f_x\,dx + f_y\,dy + f_z\,dz$ $\boxed{+1{,}000}$

$3(\underbrace{x^3 - 2y^4 + 4z^5}_{10})^2 3x^2\,dx + 3(\quad)^2(-8y^3)dy + 3(\quad)^2(20z^4)dz$

$-24 \cdot 100$ $(60)(100)$

$900(4)\,dx - 2{,}400\,(1)\,dy + 6{,}000\,(1)\,dz + 1{,}000$

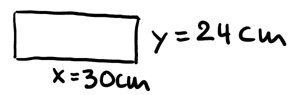
$3600\left(-\frac{3}{100}\right) - 2400\left(-\frac{1}{10}\right) + 6{,}000\,\frac{1}{100} + 1000$

$\frac{-10{,}800 + 24{,}000 + 6{,}000 + 100{,}000}{100}$

$\frac{140{,}800}{100} = 1{,}408$

Problem 16. The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error in measurement of 0.1 cm in both. Use differentials to approximate the maximum error in the calculated area of the rectangle.

$dx = dy = \frac{1}{10}$ cm



$A = xy$

$$dA = A_x\, dx + A_y\, dy =$$
$$= y\, dx + x\, dy =$$
$$= 24 \frac{1}{10} + 30 \frac{1}{10} =$$
$$= 2.4 + 3 = 5.4 \text{ cm}^2$$