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Math 251 – Spring 2023
"Week-in-Review"

Wir 1: 12.1 to 12.3

Problem 1. Find the center and radius of the sphere $x^2 + y^2 + z^2 + 4x - 2y - 8z = 5$. Does this sphere intersect the xz plane? If so, what is the intersection?

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$$\underbrace{x^2 + 4x + 4}\ \underbrace{+ y^2 - 2y + 1}\ + \underbrace{z^2 - 8z + 16} = 5 + 4 + 1 + 16$$

$$(x+2)^2 + (y-1)^2 + (z-4)^2 = 26$$

$C(-2, 1, 4)$ $r = \sqrt{26}$.

xz-plane $\Rightarrow y = 0$

$$(x+2)^2 + (0-1)^2 + (z-4)^2 = 26$$

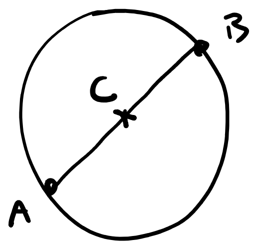
$$(x+2)^2 + (z-4)^2 + 1 = 26$$

$$\boxed{(x+2)^2 + (z-4)^2 = 25, \ y = 0}$$ intersection.

Problem 2. Find equation of the sphere with center $(1, 2, 5)$ that touches the xy plane.

$$(x-1)^2 + (y-2)^2 + (z-5)^2 = 5^2$$

Problem 3. Find the equation of the sphere if one of ~~their~~ its diameters has endpoints $\underbrace{(5,1,5)}_{A}$ and B: $(7,3,9)$.



$$r = \frac{1}{2}\,|\overline{AB}| = \frac{1}{2}\sqrt{(7-5)^2+(3-1)^2+(9-5)^2} =$$

$$= \frac{1}{2}\sqrt{4+4+16} = \frac{1}{2}\sqrt{24} = \frac{1}{\cancel{2}}\cdot\cancel{2}\sqrt{6}$$

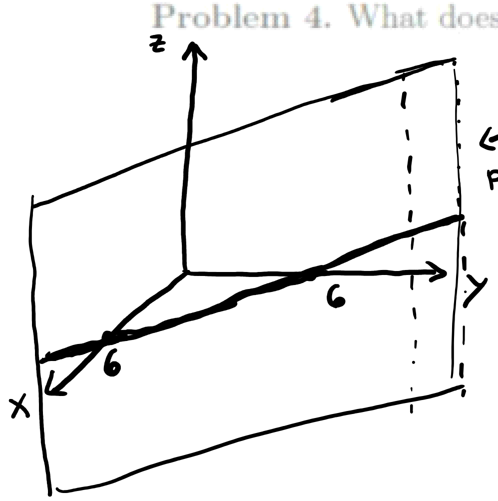
$2 \cdot 12$
$2 \cdot 2 \cdot 2 \cdot 3$

$$r = \sqrt{6}$$

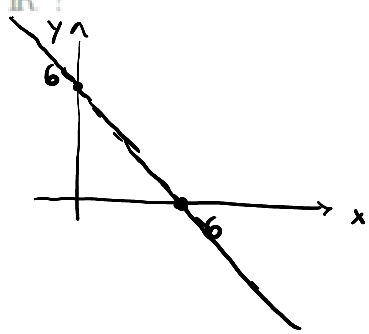
$$C = \frac{A+B}{2} = \left(\frac{12}{2}, \frac{4}{2}, \frac{14}{2}\right) = (6,2,7)$$

$$(x-6)^2 + (y-2)^2 + (z-7)^2 = 6 .$$

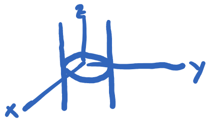
Problem 4. What does $y = 6 - x$ represent in $\mathbb{R}^3$?



plane $y = 6 - x$.



$x^2+y^2=1$

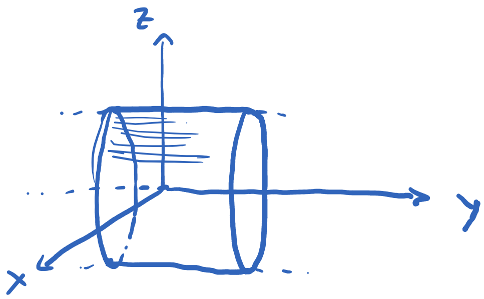


Problem 5. What does $\underbrace{x^2+z^2=16}$ represent in $\mathbb{R}^3$?

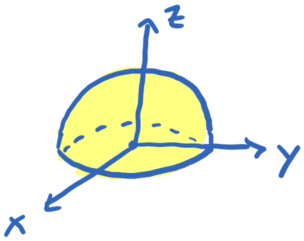
cylinder

radius $= 4$

axis y-axis



Problem 6. Write a set of inequalities that describes the solid upper hemisphere $\underbrace{x^2+y^2+z^2=9}$.



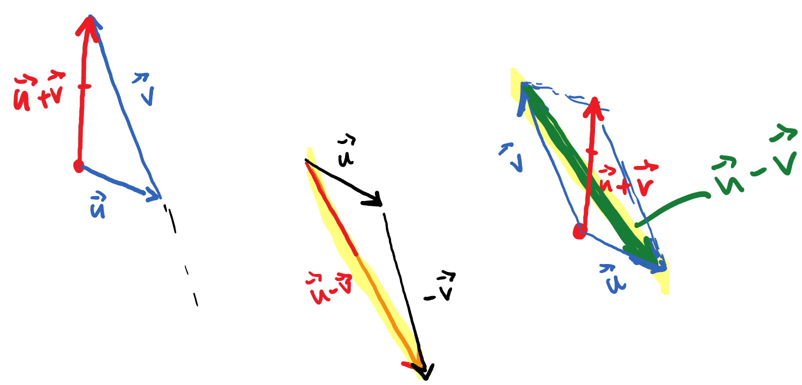
$$\begin{cases} x^2+y^2+z^2 \le 9 \\ z \geq 0 \end{cases}$$

Section 12.2

Problem 7. Give a graphical interpretation of vector sum and vector difference.

Triangle Law

Parallelogram Law



$\vec{u}-\vec{v}$

$\vec{u}+(-\vec{v})$

Problem 8. Given $\mathbf{a} = \langle -7, 1, 2\rangle$ and $\mathbf{b} = \langle 5, -1, 1\rangle$, find a unit vector in the direction of $\mathbf{a} + 2\mathbf{b}$.

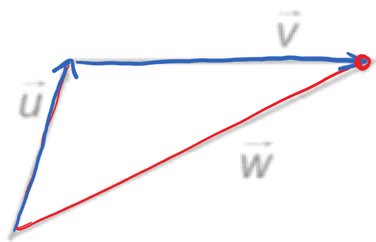
$$\vec{a} + 2\vec{b} = \langle -7, 1, 2\rangle + \langle 10, -2, 2\rangle = \underbrace{\langle 3, -1, 4\rangle = \vec{v}}$$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \qquad\qquad |\vec{v}| = \sqrt{9+1+16} = \sqrt{26}$$

$$= \frac{1}{\sqrt{26}}\langle 3, -1, 4\rangle$$

$$= \left\langle \frac{3}{\sqrt{26}}, \frac{-1}{\sqrt{26}}, \frac{4}{\sqrt{26}} \right\rangle.$$

Problem 9. For the picture seen below, write **v** in terms of **u** and **w**.



$\vec{w} = \vec{u} + \vec{v}$

$\vec{v} = \vec{w} - \vec{u}$ ✓

Section 12.3

Problem 10. Compute $\mathbf{a} \cdot \mathbf{b}$ if

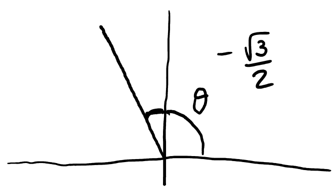
a.) $\mathbf{a} = \langle 4, 5, -1 \rangle$ and $\mathbf{b} = \langle 2, 1, 3 \rangle$. $= 8 + 5 - 3 = 10$

b.) $|\mathbf{a}| = 2$, $|\mathbf{b}| = 5$ and $\theta = 120^\circ$. $\vec{a} \cdot \vec{b} = 2 \cdot 5 \left(-\frac{1}{2}\right) = -5$

c.) $|\mathbf{a}| = 6$, $|\mathbf{b}| = 4$ and $\mathbf{a}$ is perpendicular to $\mathbf{b}$. $\vec{a} \cdot \vec{b} = 0$

d.) $|\mathbf{a}| = 6$, $|\mathbf{b}| = 4$ and $\mathbf{a}$ is parallel to $\mathbf{b}$. $\vec{a} \cdot \vec{b} = 6 \cdot 4 \cos 0 = 24$.

$\Longrightarrow$



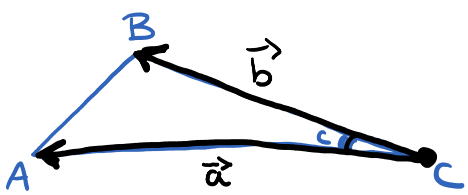
Problem 11. Are the vectors $\underbrace{-8\mathbf{i}+4\mathbf{j}+12\mathbf{k}}_{\vec{a}}$ and $\underbrace{6\mathbf{i}-3\mathbf{j}-9\mathbf{k}}_{\vec{b}}$ parallel, perpendicular, or neither?

perpendicular ? $\vec{a}\cdot\vec{b} = -48-12-108 \neq 0 \Rightarrow$ not $\perp$

$$\frac{\cancel{6}^{\,3}}{\cancel{-8}_{\,4}} = \frac{-3}{4} \qquad \frac{-\cancel{9}^{\,3}}{\cancel{12}_{\,4}}$$

$$\vec{a} \parallel \vec{b} \qquad \vec{a} = \frac{6}{-8}\vec{b} = -\frac{3}{4}\vec{b}$$

Problem 12. The points $A(0,-1,6)$, $B(2,1,-3)$ and $C(5,4,2)$ form a triangle. Find $\angle C$.



$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos C \Rightarrow \cos C = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} =$$

$$\vec{a} = \vec{CA} = A - C = \langle -5, -5, 4\rangle$$
$$\vec{b} = \vec{CB} = B - C = \langle -3, -3, -5\rangle$$

$$= \frac{15+15-20}{\sqrt{25+25+16}\,\sqrt{9+9+25}} =$$

$$= \frac{10}{\sqrt{66}\,\sqrt{33}} = \frac{10}{33\sqrt{2}}$$

$$\sqrt{6}\sqrt{11}\sqrt{3}\sqrt{11} = \sqrt{18}\;11 = 33\sqrt{2}.$$

$$c = \arccos\left(\frac{10}{33\sqrt{2}}\right).$$

Problem 13. Find the vector and scalar projection of $\underbrace{\langle 1, 2, 5\rangle}_{\vec{u}}$ onto $\underbrace{\langle 0, 7, 4\rangle}_{\vec{v}}$.

$$\text{comp}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = \frac{0+14+20}{\sqrt{0+49+16}} = \frac{34}{\sqrt{65}}$$

$$\text{proj}_{\vec{v}}\vec{u} = \frac{34}{\sqrt{65}}\cdot\frac{\vec{v}}{|\vec{v}|} = \frac{34}{\sqrt{65}}\ \frac{\langle 0,7,4\rangle}{\sqrt{65}} =$$

$$= \frac{34}{65}\langle 0, 7, 4\rangle = \left\langle 0, \frac{238}{65}, \frac{136}{65}\right\rangle.$$