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Math 251 – Spring 2023
Week-in-Review

Wir 11: Chapter 16: 16.1-16.9

Problem 18. Using the The Divergence Theorem, find the flux of $\mathbf{F} = \left\langle ye^{z^2},\ ze^{x},\ 2z+8 \right\rangle$ across S, where S is the surface of the solid bounded by the cylinder $x^2+y^2=9$, $z=0$ and $z=y-4$.

Problem 17. Using the The Divergence Theorem to evaluate $\iint\limits_S \mathbf{F} \cdot d\mathbf{S}$, where

$\mathbf{F} = \left\langle 4x,\ \sin(e^z),\ \sqrt{x^3 + y^2} \right\rangle$ and S is the surface bounded by $x^2 + y^2 = 4$, $z = 2$, $z = 4$.

Problem 16. Use Stokes' Theorem evaluate $\iint\limits_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = \langle x^2 \sin(z-5), y^2, xy \rangle$ and S is the part of the paraboliod $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$, oriented upward.

Problem 15. Set up but do not evaluate the integral which is the result of using Stokes' Theorem to find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle\, 2xz,\ 4x^2,\ 5y^2 \rangle$ and C is curve of intersection of the plane $z = x + 4$ and the cylinder $x^2 + y^2 = 4$, oriented counterclockwise when viewed from above.

Problem 14. Use Stokes' Theorem to set up but **not evaluate** $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle xz, 2xy, 3xy \rangle$ and where C is the boundary curve of the part of the plane $3x + y + z = 3$ in the first octant.
Note: Your limits of integration must be defined with the appropriate differential.

Problem 13. Find the flux of $\mathbf{F} = \langle\, x,\ y,\ -z\rangle$ across S, where S is the part of the paraboloid $z = 4 - x^2 - y^2$ that is above the xy-plane. Use the positive (outward) orientation.

Problem 12. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle\, y,\ x,\ z \rangle$ and S is the part of the paraboloid $z = x^2 + y^2$ between the planes $z = 1$ and $z = 4$.

Problem 11. Set up but do not evaluate an integral which gives is the correct set up in order to evaluate $\iint_S yz\,dS$ where S is the part of the sphere $x^2 + y^2 + z^2 = 16$ that lies between the planes $z = 2$ and $z = 2\sqrt{3}$. Note: If we parameterize the sphere $x^2 + y^2 + z^2 = \rho^2$ by $\mathbf{r}(\theta, \phi) = \langle \rho \sin(\phi)\cos(\theta),\ \rho\sin(\phi)\sin(\theta),\ \rho\cos(\phi)\rangle$, then $|\mathbf{r}_\theta \times \mathbf{r}_\phi| = \rho^2 \sin(\phi)$.

Problem 10. Find the surface area of the part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$.

Problem 9. Find the surface area of the part of the plane $6x + 2y + 8z = 24$ in the first octant.

Problem 8. Given $\mathbf{F} = \langle 4xe^z, \cos(y), 2x^2e^z \rangle$ and $\mathbf{r}(t) = \langle \sin(t), t, \cos(t) \rangle$, compute $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ for $0 \leq t \leq \dfrac{\pi}{2}$.

Problem 7. Find the work done by the force field $\mathbf{F} = \langle x^2,\ y^2 \rangle$ in moving a particle along the arc of the parabola $y = 2x^2$ from the point $(-2, 8)$ to $(1, 2)$.

Problem 6. A particle starts at the point $(-3, 0)$, moves along the x-axis to the point $(3, 0)$, then along the semicircle $y = \sqrt{9 - x^2}$, then back to the starting point. Find the work done on this particle by the force field $\mathbf{F} = \langle 3x, x^3 + 3xy^2 \rangle$.

Problem 5. Evaluate $\int_C (xy)\ dx + (x-y)dy$, where C is the line segment from $(1,1)$ to $(2,0)$ and then from $(2,0)$ to $(3,5)$.

Problem 4. Find $\int_C (3y + 7e^{\sqrt{x}})dx + (8x + 9\cos(y^2))dy$, where C is the boundary of the region enclosed by $y = x^2$ and $x = y^2$.

Problem 3. Evaluate $\int_C z\,dx + (xy)\,dy$, where C is the line segment from $(-1, 1, 0)$ to $(1, 2, 0)$.

Problem 2. Find $\int_C x ds$, where C is the right half of the circle $x^2 + y^2 = 4$, oriented counter-clockwise.

Problem 1. Evaluate $\int_C y\,ds$, where C is parameterized by $\mathbf{r}(t) = \langle\, t,\ t^3\rangle$, $0 \le t \le 1$.