EXAM 3 REVIEW

Exercise 1

Convert $(x, y, z) = (\sqrt{3}, -1, 2\sqrt{3})$ from rectangular to cylindrical and spherical coordinates.

Exercise 2

Convert $(r, \theta, z) = (1, 2\pi/3, -3)$ to rectangular coordinates.

Exercise 3

Convert $(\rho, \theta, \phi) = (6, \pi/3, \pi/6)$ to rectangular coordinates.

Exercise 4

In cylindrical coordinates, graph $z = r$. Also graph $r = \cos(\theta)$.

Exercise 5

In spherical coordinates, graph $\theta = \pi/3$. Also graph $\phi = 2\pi/3$.

Exercise 6

Set up but do not evaluate the following integrals. Set them up in the coordinate system you would try to actually evaluate them in.

(a) $\iint_D \frac{y}{x^2+1}\,\mathrm{d}A$, where $D = \{(x, y) : 0 \le x \le 4, 0 \le y \le \sqrt{x}\}$.

(b) $\iint_D x\cos(y)\,\mathrm{d}A$, where D is the region bounded by $y = 0$, $y = x^2$, and $x = 1$.

Exercise 6 continued on next page...

(c) $\displaystyle\iiint_E xe^{x^2+y^2+z^2}\,\mathrm{d}V$, where E is the portion of the unit ball in the first octant.

(d) Find the area of the region inside the circle $(x-1)^2+y^2=1$ and outside the circle $x^2+y^2=1$.

Exercise 6 continued on next page...

(e) Find the volume under $z = x^2 + 3y$ and above the triangle in the xy-plane with vertices $(0, 0)$, $(2, 0)$, and $(-1, 0)$.

(f) Find the volume of the region bounded between the paraboloid $z = 1 + 2x^2 + 2y^2$ and the plane $z = 7$ in the first octant.

Exercise 6 continued on next page. . .

(g) $\iiint_E y^2x^2 \, \mathrm{d}V$, where E lies below the cone $\phi = 5\pi/6$ and between the spheres $\rho = 1$ and $\rho = 3$.

(h) Find the volume of the region that is inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 4y^2 + z^2 = 64$.

Exercise 6 continued on next page...

(i) $\iiint_E (x^2+y^2+z^2)^{3/2}\,\mathrm{d}V$, where E lies above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=16$.

(j) $\iiint_E (x+y+z)\,\mathrm{d}V$, where E is the solid under the paraboloid $z=4-x^2-y^2$, above the xy-plane, and on the positive x side of the plane $x=0$.

Exercise 7

Evaluate the following.

(a) $\int_0^1 \int_{x^2}^1 \sqrt{y}\sin(y)\,\mathrm{d}y\,\mathrm{d}x$

(b) $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}\,\mathrm{d}y\,\mathrm{d}x$

Exercise 8

Find the mass of a ball of radius 3 if its density is proportional to the distance from the center of the ball. (Assume that the constant of proportionality is 1.) [Note: not all instructors covered this topic.]