EXAM 2 REVIEW

Exercise 1

Find the domain and range of the following functions.

(a) $f(x,y) = \sqrt{x^2 + y^2 - 9}$

(b) $g(x,y) = \sqrt{y^2 - 4} + \sqrt[4]{y + x^2}$

Exercise 2

Draw some level curves of the following functions. Draw the gradient at several points on the graph of level curves.

(a) $f(x, y) = \ln(xy)$

(b) $g(x, y) = \frac{y}{x^2+y^2}$

Exercise 3

The average energy E (in kcal) needed for a lizard to walk or run a distance of 1km has been modeled by the equation

$$E(m, v) = \frac{8}{3}m^{2/3} + \frac{7m^{3/4}}{2v},$$

where m is the body mass of the lizard (in grams) and v is the speed (in km/h). Compute $E_m(400, 8)$ and $E_v(400, 8)$ and interpret your results.

Exercise 4

In a study of frost depth, it was found that the temperature T at time t (in days) at a depth x (in feet) can be modeled by the function

$$T(x, t) = T_0 + T_1 e^{-\lambda x} \sin(\frac{2\pi}{365} - \lambda x),$$

where λ, T_0, and T_1 are some constants.

(a) Compute $\frac{\partial T}{\partial x}$. What is its physical significance?

(b) Compute $\frac{\partial T}{\partial t}$. What is its physical significance?

Exercise 5

Find the tangent plane to $z = \dfrac{x}{y^2}$ at the point $(-4, 2, -1)$.

Exercise 6

Use differentials to estimate the amount of metal in a closed cylindrical can that is 12cm high and 8cm in diameter if the tin is 0.04cm thick.

Exercise 7

Use differentials to estimate the value of $\ln((1.1)^3 + (1.2)^2)$.

Exercise 8

Recall the ideal gas law: $PV = nRT$. ($R = 8.31\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2\cdot\text{K}\cdot\text{mol}}$) Suppose we have a closed box of 2 moles of a gas. If we increase the temperature according to $T(t) = (200 + t^2)$ K and change the volume according to $V(t) = (10 - t)\,\text{m}^3$, how fast is the pressure changing at time $t = 3$?

Exercise 9

The relative humidity can be expressed as the formula

$$R(P,T,w) = \frac{w}{a + bPe^{\frac{T}{1+T}}},$$

where P is the pressure, T is the temperature, and w is the amount of water in the air. The heat index is a function of R and T. Find an expression for how fast the heat index changes as the temperature is increased in a room of volume V with a fixed amount of water in the air.

Exercise 10

Compute the gradient of $f(x,y,z) = x^2\sin(yz) + 2y^2z$.

Exercise 11

Compute the directional derivative of $g(x, y) = e^{xy^2}$ at the point $(0, 2)$ in the direction $\langle -4, 3 \rangle$.

Exercise 12

At what point on the ellipsoid $x^2+y^2+2z^2 = 1$ is the tangent plane parallel to the plane $x+2y+z = 1$?

Exercise 13

The temperature at a point (x, y, z) is given by

$$T(x, y, z) = 200e^{-x^2-3y^2-9z^2},$$

where T is measured in °C and x, y, z are measured in meters.

(a) Find the rate of change of temperature at the point $P(2, -1, 2)$ in the direction toward the point $(3, -3, 3)$.

(b) In which direction does the temperature increase fastest at P?

(c) Find the maximum rate of increase at P.

Exercise 14

Find the shortest distance from $(2, 0, -3)$ to the plane $x + y + z = 1$.

Exercise 15

Find the absolute min and max of $g(x, y) = xy^2$ on the region $R = \{(x, y) : x \geq 0, x^2 + y^2 \leq 3\}$.

Exercise 16

Using Lagrange multipliers, find the max and min of $f(x, y) = 2x + 2y + z$ subject to the constraint $x^2 + y^2 + z^2 = 9$.