EXAM 1 REVIEW

Exercise 1

Describe the following regions of $\mathbb{R}^3$ in words.

(a) $z > 0$

(b) $x = y$

(c) $x^2 + y^2 = 4$

(d) $(x-1)^2 + (y-2)^2 + (z+1)^2 \leq 9$

(e) $1 \leq z^2 + x^2 \leq 9$

Exercise 2

What is the intersection of the following regions in $\mathbb{R}^3$?

(a) $x \geq 0$ and $x^2 + y^2 + z^2 \leq 4$

(b) $x^2 + z^2 = 1$ and $y = 2$

(c) $x^2 + z^2 = 4$ and $z = 1$

(d) $1 \leq x^2 + y^2 + z^2 \leq 9$ and $y = 2$

Exercise 3

P, Q, and R form a triangle. What is $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RP}$?

Exercise 4

Let $\mathbf{u} = \langle 3, -1, 2 \rangle$. Find a vector $\mathbf{v}$ such that $\mathbf{v}$ goes in the direction of $\langle 1, 2, -2 \rangle$ and $\text{comp}_{\mathbf{u}}\mathbf{v} = -4$.

Exercise 5

Is the triangle formed by the vertices $A(1,2,3)$, $B(5,1,6)$, $C(3,4,1)$ a right triangle?

Exercise 6

Find the vector projection of $\overrightarrow{AB}$ onto $\overrightarrow{BC}$.

Exercise 7

Find a unit vector perpendicular to the plane containing the point $(-2, 5, 2)$ and the line $L(t) = \langle 2t, 3+t, 1-2t\rangle$.

Exercise 8

Let V be the parallelepiped whose edges all have length 2. One side of V lies in the xy-plane. The angle between the edges that lie in the xy-plane is $45°$. An edge of V that is not in the xy-plane makes a $30°$ angle with the z-axis. What is the volume of V?

Exercise 9

Find the point on the line $L(t) = \langle 2+t, 2-2t, -1+t \rangle$ that is closest to the point $(-4, 1, 5)$.

Exercise 10

What is the domain of $\mathbf{r}(t) = \langle \sqrt{2t+4}, \ln(3-t), (1-t)^{-1} \rangle$?

Exercise 11

Find parametric and symmetric equations for a line that is perpendicular to the plane $3x-7y+4z=8$ and passes through the point $(5,1,-4)$.

Exercise 12

Determine if the following pair of lines is intersecting, parallel, or skew: $L_1(t)=\langle 1+2t,-2-t,3+2t\rangle$ and $L_2(t)=\langle 1+t,-2+3t,1+2t\rangle$.

Exercise 13

Find a plane whose intercepts with the x, y, and z axes are 3, 7, and -2, respectively.

Exercise 14

Find the intersection between the plane $x + y = z$ and the plane $3x - 2y - 2z = 5$.

Exercise 15

Draw the traces of the equation $x^2 - 3y^2 + z^2 = 4$. What shape is it?

Exercise 16

Draw the traces of the equation $3x^2 - y^2 - 2z^2 = 0$. What shape is it?

Exercise 17

Find the intersection between the curve $\mathbf{r}(t) = \langle t^2, \cos(t), \sin(t)\rangle$ and the surface $3x^2+2y^2+2z^2 = 5$.

Exercise 18

Draw the projection of $\mathbf{r}(t) = \langle t^2, t, \cos(t)\rangle$ onto the xy and yz planes.

Exercise 19

Find the line tangent to $\mathbf{r}(t) = \langle t^2, t, \cos(\pi t)\rangle$ at the point $(4, -2, 1)$.

Exercise 20

What is $\int_1^2 \big(\sin(\pi t)\mathbf{i} + e^{2t}\mathbf{j} - 7\mathbf{k}\big)\,\mathrm{d}t$?

Exercise 21

Find the length of the curve $\mathbf{r}(t) = \langle \sin(\pi t), 3t, \cos(\pi t) \rangle$ from $(0, 0, 1)$ to $(0, 6, 1)$.