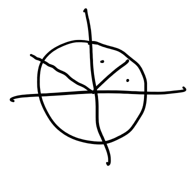
Wir 7: Sections 15.1, 15.2, 15.3

Section 15.1

Problem 1. Find $\int_0^{\pi/4} x \sin(3y)\, dy$

$$-x\,\frac{1}{3}\cos(3y)\Big|_{y=0}^{y=\frac{\pi}{4}} = -\frac{x}{3}\left[\cos\frac{3\pi}{4} - \cos 0\right] =$$



$$= -\frac{x}{3}\left[-\frac{\sqrt{2}}{2} - 1\right] =$$

$$= \frac{x}{3}\left(\frac{\sqrt{2}+2}{2}\right)$$

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Problem 2. Find $\int_1^e \frac{y\ln(x)}{x}\, dx$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$$\int_{\ln 1}^{\ln e} y\,u\,du = y\left[\frac{u^2}{2}\right]_0^1 = y\left(\frac{1}{2} - 0\right) = \frac{y}{2}.$$

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Problem 3. Evaluate $\int^2 \int^3 (xy + x + y)\, dy\, dx$ and $\int^3 \int^2 (xy + x + y)\, dx\, dy$

Problem 3. Evaluate $\int_0^2 \left\{ \int_0^3 (xy+x+y)\,dy \right\} dx$ and $\int_0^3 \int_0^2 (xy+x+y)\,dxdy$

$$xy^2\frac{1}{2} + xy + \frac{y^2}{2}\Big]_{y=0}^{3}$$

$$\frac{9}{2}x + 3x + \frac{9}{2} - \phi$$

$$\int_0^2 \left(\frac{15}{2}x + \frac{9}{2}\right) dx =$$

$$= \frac{15}{2}\frac{x^2}{2} + \frac{9}{2}x\Big|_0^2 =$$

$$= \frac{15}{\cancel{4}}\cdot\cancel{4} + \frac{9}{\cancel{2}}\cdot\cancel{2} - \phi$$

$$= 24 \checkmark$$

$$\frac{x^2}{2}y + \frac{x^2}{2} + yx\Big]_{x=0}^{2} =$$

$$\frac{\cancel{4}^{\,2}}{\cancel{2}}y + \frac{4}{2} + 2y - \phi$$

$$\int_0^3 (4y+2)\,dy =$$

$$= 2y^2 + 2y\Big|_0^3 =$$

$$= 2\cdot 9 + 2\cdot 3 =$$

$$= 18 + 6 = 24 \checkmark$$

Fubini's Theorem: If f is continuous on the rectangle $R = [a,b]\times[c,d]$, then

(1) $\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dydx = \int_c^d \int_a^b f(x,y)\,dxdy.$

(2) In the case where $f(x,y) = g(x)h(y)$, then

$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d g(x)h(y)\,dydx = \int_a^b g(x)\,dx \int_c^d h(y)\,dy$$

Problem 4. Find $\iint_R \frac{x}{y^2}\,dA$, where $R = [0,4]\times[1,2]$

$$\int_0^4 \left\{ \int_1^2 \frac{x}{y^2}\,dy \right\} dx \qquad D\left(\frac{-1}{y}\right) = \frac{+1}{y^2}$$

$$-\frac{x}{y}\Big]_{y=1}^{y=2} = -x\left(\frac{1}{2} - 1\right) = \frac{1}{2}x$$

$$\int_0^4 \frac{1}{2}x\,dx = \frac{1}{2}\frac{x^2}{2}\Big|_0^4 = \frac{16}{4} - 0 = 4$$

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Problem 5. Find $\iint_R x\sec^2 y\,dA$, where $R=\{(x,y)|0\le x\le 2, 1\le y\le \frac{\pi}{4}\}$

$$\int_0^2 x\,dx \cdot \int_1^{\pi/4} \sec^2 y\,dy = \frac{x^2}{2}\Big|_0^2 \cdot \tan y\Big|_1^{\pi/4} =$$

$$= \left(\frac{4}{2}-0\right)(1-\tan 1) =$$

$$= 2(1-\tan 1).$$

Problem 6. Find $\iint_R e^{2x+y}\,dA$, where $R=\overbrace{[0,\ln 2]}^{x}\times\overbrace{[0,\ln 3]}^{y}$

$$\iint e^{2x}\cdot e^{y}\,dA$$

$$\int_0^{\ln 2} e^{2x}\,dx \cdot \int_0^{\ln 3} e^{y}\,dy = \frac{1}{2}e^{2x}\Big|_0^{\ln 2} \cdot e^{y}\Big|_0^{\ln 3} =$$

$$= \frac{1}{2}\left(e^{2\ln 2}-1\right)\left(e^{\ln 3}-1\right) = \frac{1}{\cancel{2}}(4-1)\cancel{(3-1)} = 3$$

$$\left(e^{\ln 2}\right)^2$$

#

Problem 7. Find $\iint_R (y\cos(xy))\,dA$, where $R=\overbrace{[0,2]}^{x}\times\overbrace{[0,\pi]}^{y}$

$$\int_0^{\pi}\int_0^{2} y\cos(xy)\,dx\,dy \qquad \cancel{y}\frac{1}{\cancel{y}}\sin(xy)\Big|_{0=x}^{2=x} =$$

$$= \sin 2y - \cancel{0}$$

$$\int^{\pi} \sin(2y)\,dy = \qquad -(\cos 2y)\frac{1}{2}\Big|_{y=0}^{y=\pi} =$$

0

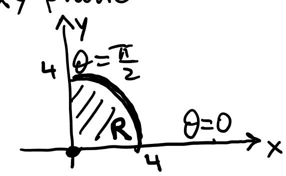
$$= -\frac{1}{2}\left[1-1\right] = \emptyset$$

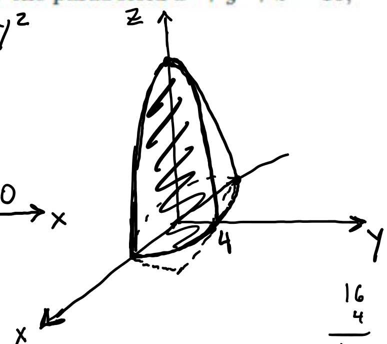
Problem 8. Find the volume of the solid S that is bounded by the paraboloid $x^2 + y^2 + z = 16$, $z = 0$, $0 \le x \le 4$, $0 \le y \le 4$.

$f(x,y)$

$z = 16 - x^2 - y^2$

xy plane





$$\text{Vol} = \iint_R f(x,y)\, dA =$$

$$= \iint_R (16 - x^2 - y^2)\, dA$$

$$\begin{array}{r} 16 \\ \times\ 4 \\ \hline 64 \end{array}$$

Polar Coord. $\displaystyle\int_0^{\pi/2}\int_0^4 (16 - r^2)\, r\, dr\, d\theta$

$$\int r - r^3\, dr$$

$$16\frac{r^2}{2} - \frac{r^4}{4}\Big|_0^4 =$$

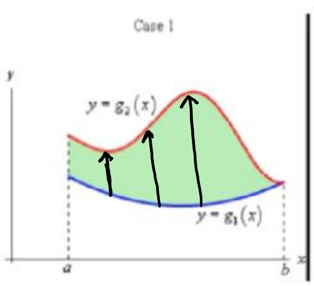
$$16 \cdot 8\ \frac{\cancel{16}}{\cancel{2}} - \frac{4^{\cancel{4}3}}{\cancel{4}}$$

$$128 - 64 = 64$$

$$\int_0^{\pi/2} 64\, d\theta = 64\,\frac{\pi}{2} = \boxed{32\pi}$$

Section 15.2

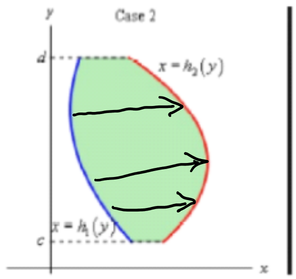
Type I: A plane region D is said to be of type I if it lies between the graphs of two continuous functions of x, that is $D = \{(x,y) | a \le x \le b, g_1(x) \le y \le g_2(x)\}$.



If f is continuous on a type I region $D = \{(x,y) | a \le x \le b, g_1(x) \le y \le g_2(x)\}$, then

$$\iint_R f(x,y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy dx$$

Type II: A plane region D is said to be of type II if it lies between the graphs of two continuous functions of y, that is $D = \{(x,y)|c \le y \le d, h_1(y) \le x \le h_2(y)\}$.



If f is continuous on a type II region $D = \{(x,y)|c \le y \le d, h_1(y) \le x \le h_2(y)\}$, then

$$\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dxdy$$

Problem 9. Evaluate $\int_1^4 \int_1^{\sqrt{x}} (x+y)\,dydx$ set up as Type I.

$$\left[xy + \frac{1}{2}y^2\right]_{y=1}^{y=\sqrt{x}} = x\sqrt{x} + \frac{1}{2}x - x - \frac{1}{2} = x^{3/2} - \frac{1}{2}x - \frac{1}{2}$$

$$\int_1^4 \left(x^{3/2} - \frac{1}{2}x - \frac{1}{2}\right)dx = \frac{2}{5}x^{5/2} - \frac{1}{2}\frac{x^2}{2} - \frac{1}{2}x\Big|_1^4 =$$

$$= \frac{2}{5}2^5 - \frac{16}{4} - 2 - \left(\frac{2}{5} - \frac{1}{4} - \frac{1}{2}\right) =$$

$$= \frac{64}{5} - 4 - 2 - \frac{2}{5} + \frac{3}{4} = \frac{62}{5} + \frac{3}{4} - 6 =$$

$$= \frac{248 + 15 - 120}{20} = \frac{143}{20}$$

do as a Type 2

$$\begin{array}{r} 128 \\ 15 \\ \hline 143 \end{array}$$

Problem 10. Evaluate $\int_0^1 \int_0^y (3 + x^2y)\,dxdy$ Type 2

$$3x + \frac{1}{3}x^3y\Big|_{x=0}^{x=y} = 3y + \frac{1}{3}y^4 - \emptyset$$

$$\int_0^1 \left(3y + \frac{1}{3}y^4\right) dy = 3\frac{y^2}{2} + \frac{1}{3}\frac{1}{5}y^5\Big|_0^1 =$$

$$= \frac{3}{2} + \frac{1}{15} = \frac{45+2}{30} = \frac{47}{30}$$

do as a Type1

$= \frac{?}{2} + \frac{\cdot}{15} = \frac{\quad}{30} \qquad 30$

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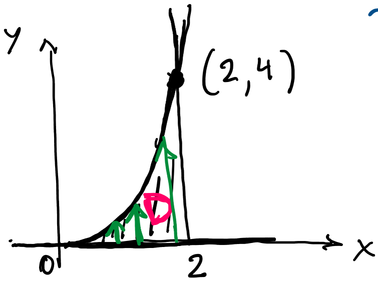
Problem 11. Sketch the region of integration and evaluate $\iint_D xe^y \, dA$ where D is the region bounded by $y = 0$, $y = x^2$ and $x = 2$

$D: \begin{cases} 0 \le x \le 2 \\ 0 \le y \le x^2 \end{cases}$



do as a Type 2

$$\int_0^2 \left\{ \int_0^{x^2} x e^y \, dy \right\} dx$$

$$x\left[e^y\right]_0^{x^2} = x\left(e^{x^2} - e^0\right) = xe^{x^2} - x$$

$$\int_0^2 \left(xe^{x^2} - x\right) dx$$

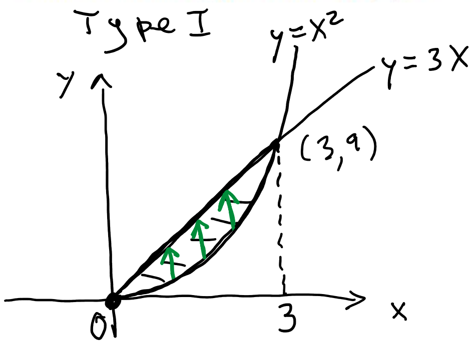
$u = x^2 \quad du = 2x\,dx$

$$\int_{0^2}^{2^2} \frac{1}{2} e^u \, du = \frac{1}{2} e^u \Big|_0^4 = \frac{1}{2}\left(e^4 - e^0\right)$$

$$\frac{1}{2}\left(e^4 - 1\right) - \left[\frac{x^2}{2}\right]_0^2 = \frac{1}{2}\left(e^4 - 1\right) - \left(\frac{4}{2} - 0\right) = \frac{1}{2}e^4 - \frac{1}{2} - 2 = \frac{e^4 - 5}{2}$$

Problem 12. Set up but do not evaluate both a type I and type II integral for $\iint_D f(x,y)\, dA$, where D is the region bounded by $y = x^2$ and $y = 3x$.

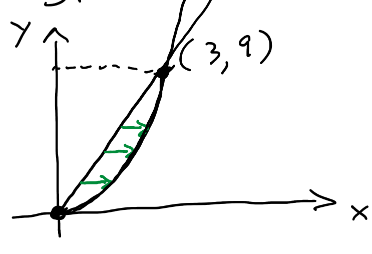
Type I



$3x = x^2 \to x = 0, \; x = 3$

$$\int_0^3 \int_{x^2}^{3x} f(x,y)\, dy\, dx$$

Type II $\quad x = \sqrt{y}, \; x = y/3$



$\begin{cases} 0 \le y \le 9 \\ \frac{y}{3} \le x \le \sqrt{y} \end{cases}$

$$\int_0^9 \int_{y/3}^{\sqrt{y}} f(x,y)\, dx\, dy$$

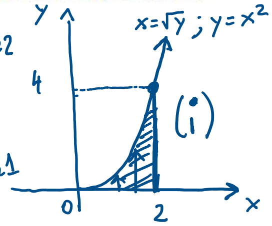
$0 \quad y/3$

Problem 13. Sketch the region of integration and change the order of integration.

(i) $\displaystyle\int_0^4 \int_{\sqrt{y}}^2 f(x,y)\,dx\,dy$

$x=\sqrt{y}$

GIVEN: $\begin{cases} \sqrt{y} \le x \le 2 \\ 0 \le y \le 4 \end{cases}$ Type2

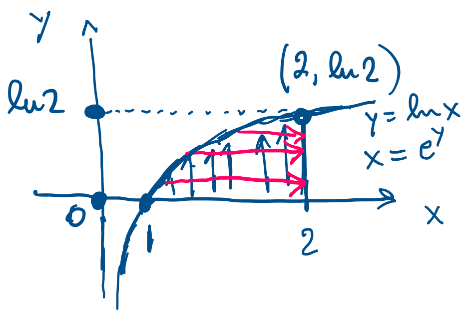


$\begin{cases} 0 \le x \le 2 \\ 0 \le y \le x^2 \end{cases}$ Type1

$$\int_0^2 \int_0^{x^2} f(x,y)\,dy\,dx$$

(ii) $\displaystyle\int_1^2 \int_0^{\ln x} f(x,y)\,dy\,dx$

GIVEN $\begin{cases} 0 \le y \le \ln x \\ 1 \le x \le 2 \end{cases}$ Type1

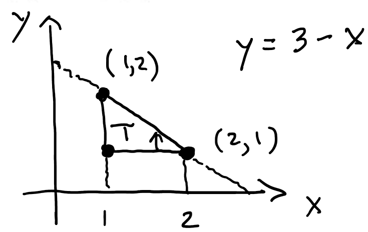


Type2 $\begin{cases} 0 \le y \le \ln 2 \\ e^y \le x \le 2 \end{cases}$

$$\int_0^{\ln 2} \int_{e^y}^{2} f(x,y)\,dx\,dy$$

Problem 14. Set up but do not evaluate a double itegral that gives the volume of the solid under the surface $z = xy$ and above the triangle with vertices $(1,1)$, $(1,2)$ and $(2,1)$

$\hookrightarrow f(x,y) = xy$



$$\text{Vol} = \iint_T f(x,y)\,dA$$

$\begin{cases} 1 \le x \le 2 \\ 1 \le y \le 3-x \end{cases}$

$$\int_1^2 \left\{ \int_1^{3-x} xy\,dy \right\} dx \qquad x\frac{y^2}{2}\Big]_{y=1}^{y=3-x} =$$

$$= \frac{x}{2}\left[(3-x)^2 - 1^2\right] = \frac{x}{2}\left(9-6x+x^2-1\right)$$

$$\frac{9x}{2} - \frac{6}{2}x^2 + \frac{x^3}{2} - \frac{x}{2}$$

$$\frac{8}{2}x = 4x$$

$$\int_1^2 \left(-3x^2 + \frac{x^3}{2} + 4x\right) dx =$$

$$= -x^3 + \frac{1}{2}\frac{x^4}{4} + 2x^2 \Big|_1^2 =$$

$$= \cancel{-8} + \cancel{\frac{16}{8}} + \cancel{8} + 1 - \frac{1}{8} \cancel{-2} = \frac{7}{8}$$

Problem 15. Evaluate $\displaystyle\int_0^2 \int_x^2 e^{-y^2}\,dy\,dx$

TypeI $\begin{cases} x \le y \le 2 \end{cases}$

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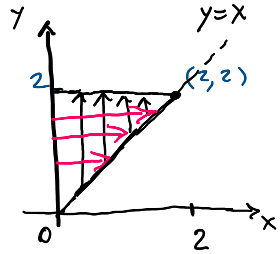
Problem 15. Evaluate $\int_0^2 \int_x^2 e^{-y^2}\,dy\,dx$

Type I $\begin{cases} x \le y \le 2 \\ 0 \le x \le 2 \end{cases}$

Type 2 $\begin{cases} 0 \le y \le 2 \\ 0 \le x \le y \end{cases}$



$$\int_0^2 \left\{ \int_0^y e^{-y^2}\,dx \right\} dy$$

$$\left[x\, e^{-y^2}\right]_{x=0}^{x=y} = y\, e^{-y^2}$$

$$\int_0^2 y\, e^{-y^2}\,dy = \qquad u = -y^2 \qquad du = -2y\,dy$$

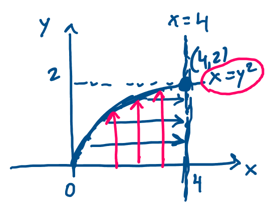
$$= \int_{-0}^{-4} -\frac{1}{2} e^u\,du = \int_{-4}^{0} \frac{1}{2} e^u\,du = \frac{1}{2} e^u \Big|_{-4}^{0} = \frac{1}{2}\left(1 - e^{-4}\right) =$$

$$= \frac{1}{2}\left(1 - \frac{1}{e^4}\right).$$

Problem 16. Evaluate $\int_0^2 \int_{y^2}^4 \sqrt{x}\sin x\,dx\,dy$

Type 2 $\begin{cases} y^2 \le x \le 4 \\ 0 \le y \le 2 \end{cases}$

Type 1 $\begin{cases} 0 \le x \le 4 \\ 0 \le y \le \sqrt{x} \end{cases}$



$$\int_0^4 \left\{ \int_0^{\sqrt{x}} \sqrt{x}\sin x\,dy \right\} dx$$

$$\left(\sqrt{x}\sin x\right)\underbrace{\left(\sqrt{x} - 0\right)}_{[y]_0^{\sqrt{x}}}$$

by parts

$$\int_0^4 x \sin x\,dx$$

$$\left[-x\cos x + \sin x\right]_0^4 =$$

$$= -4\cos 4 + \sin 4 - (\phi)$$

$$\sin 4 - 4\cos 4$$

\#

Tabular Integration

der		int.
x	$+$	$\sin x$
1	$-$	$-\cos x$
0		$-\sin x$

Section 15.3

Recall: If $P(x, y)$ is a point in the xy-plane, we can represent the point P in polar form: Let r be the distance from O to P and let θ be the angle between the polar axis and the line OP. Then the point P is represented by the ordered pair (r, θ), and r, θ are called the **polar coordinates** of P.

Connecting polar coordinates with rectangular coordinates:

a.) $x = r\cos(\theta)$, $y = r\sin(\theta)$

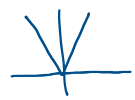
b.) $\tan(\theta) = \dfrac{y}{x}$, thus $\theta = \arctan\left(\dfrac{y}{x}\right)$.

c.) $x^2 + y^2 = r^2$

Problem 1. Find the cartesian coordinates of the polar point $\left(\overset{r}{2}, \overset{\theta}{\dfrac{2\pi}{3}}\right)$.

$x = 2\cos\frac{2\pi}{3} = 2\left(-\frac{1}{2}\right) = -1$

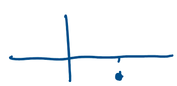
$y = 2\sin\frac{2\pi}{3} = 2\,\frac{\sqrt{3}}{2} = \sqrt{3}$



$$\boxed{(-1, \sqrt{3})}$$

Problem 2. Find the polar coordinates of the rectangular point $(\sqrt{3}, -1)$.

$r = \pm\sqrt{3+1} = \pm 2$



$\theta = \tan^{-1}\frac{-1}{\sqrt{3}} = -\tan^{-1}\frac{1}{\sqrt{3}} = -\frac{\pi}{6}$

$\left(2, -\frac{\pi}{6}\right)$

$\left(-2, \frac{5\pi}{6}\right)$

.

Problem 3. Find a cartesian equation for the curve described by $r = 2\sin\theta$.

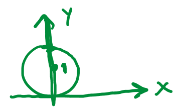
$r = 2\sin\theta$

$r^2 = 2r\sin\theta$

$\boxed{x^2+y^2 = 2y}$

circle with center

$c(0,1)$ $r=1$



$x^2+y^2-2y+1 = 0+1$

Problem 4. Find a polar equation for $y = 1 + 3x$

$r\sin\theta = 1 + 3r\cos\theta$

$r(\sin\theta - 3\cos\theta) = 1$

$r = \dfrac{1}{\sin\theta - 3\cos\theta}$

Change to Polar Coordinates in a Double Integral: If f is continuous on a polar rectangle R given by $0 \le a \le r \le b$, $\alpha \le \theta \le \beta$, where $0 \le \beta - \alpha \le 2\pi$, then

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

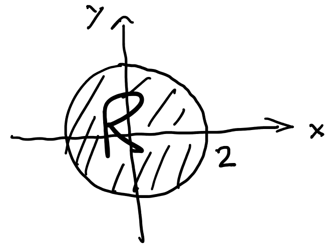
Problem 5. Evaluate $\iint_R (x+2)\,dA$, where R is the region bounded by the circle $x^2+y^2=4$.

2π 2 y

Problem 5. Evaluate $\iint_R (x+2)\,dA$, where R is the region bounded by the circle $x^2+y^2=4$.

$$R: \begin{cases} 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \end{cases}$$

$$\int_0^{2\pi}\int_0^2 (r\cos\theta + 2)\, r\, dr\, d\theta$$



$$\int_0^2 (r^2\cos\theta + 2r)\, dr$$

$$\left[\frac{1}{3}r^3\cos\theta + r^2\right]_{r=0}^{r=2} =$$

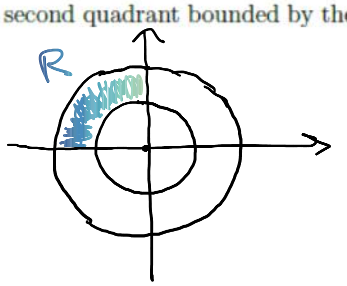
$$= \frac{8}{3}\cos\theta + 4 - 0$$

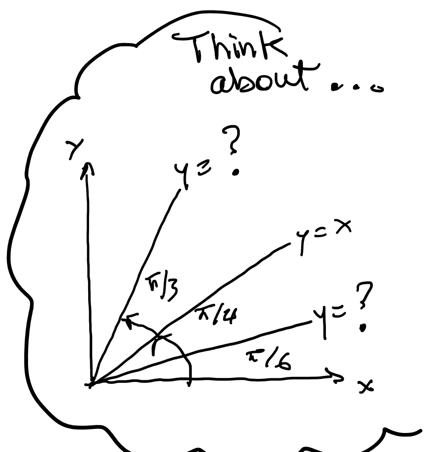
$$\int_0^{2\pi}\left(4 + \frac{8}{3}\cos\theta\right) d\theta =$$

$$= \left[4\theta + \frac{8}{3}\sin\theta\right]_0^{2\pi} = 8\pi + 0 - 0$$

$$\boxed{8\pi}$$

Problem 6. Evaluate $\iint_R 4y\,dA$, where R is the region in the second quadrant bounded by the circles $x^2+y^2=1$ and $x^2+y^2=4$.





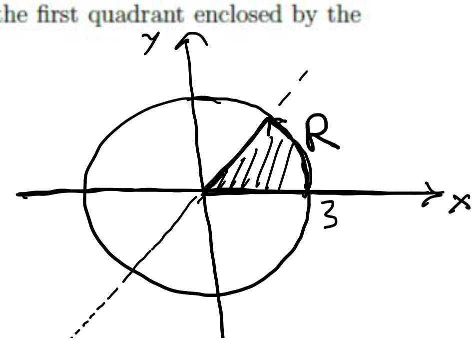
$$R: \begin{cases} 1 \le r \le 2 \\ \frac{\pi}{2} \le \theta \le \pi \end{cases}$$

$$\int_{\pi/2}^{\pi}\int_1^2 4 r \sin\theta\, r\, dr\, d\theta =$$

$$= 4\cdot\int_{\pi/2}^{\pi}\sin\theta \cdot \int_1^2 r^2\, dr = 4\left[-\cos\theta\right]_{\pi/2}^{\pi}\cdot\left[\frac{1}{3}r^3\right]_1^2 =$$

$$= -4(-1 - 0)\left(\frac{8}{3} - \frac{1}{3}\right) = 4\cdot\frac{7}{3} = \frac{28}{3}$$

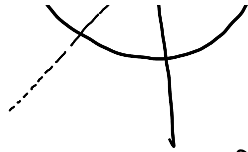
Problem 7. Evaluate $\iint_R 3x^2\,dA$, where R is the region in the first quadrant enclosed by the by the circle $x^2+y^2=9$ and the lines $y=0$ and $y=x$.



$$R: \begin{cases} 0 \le r \le 3 \\ 0 \le \theta \le \frac{\pi}{4} \end{cases}$$

$$\int_0^{\pi/4}\int_0^3$$

$$\int_0^{\pi/4}\int_0^3 \underbrace{3r^2}\cos^2\theta\, \underbrace{r}\,dr\,d\theta$$



$$\int_0^3 3r^3\,dr = \frac{3}{4}r^4\Big|_0^3 = \frac{3^5}{4}$$

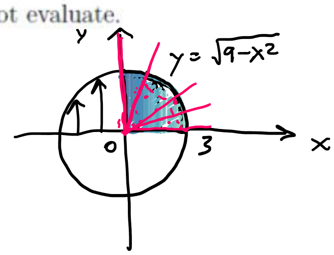
$$\frac{3^5}{4}\int_0^{\pi/4}\cos^2\theta\,d\theta = \frac{3^5}{4}\int_0^{\pi/4}\frac{1}{2}(1+\cos 2\theta)\,d\theta = \frac{3^5}{2^3}\left[\theta+\frac{1}{2}\sin 2\theta\right]_0^{\pi/4} =$$

$$= \frac{3^5}{2^3}\left[\frac{\pi}{4}+\frac{1}{2}1-\emptyset\right] = \frac{3^5}{8}\left(\frac{\pi}{4}+\frac{1}{2}\right).$$

Problem 8. Change $\int_0^3\int_0^{\sqrt{9-x^2}} x^2\,dydx$ to a polar double integral. Do not evaluate.

$$\begin{cases} 0\le x\le 3 \\ 0\le y\le \sqrt{9-x^2}\end{cases}$$

$$\begin{cases} 0\le r\le 3 \\ 0\le \theta\le \frac{\pi}{2}\end{cases}$$



$$\int_0^{\pi/2}\int_0^3 r^2\cos^2\theta\; r\,dr\,d\theta$$

Problem 9. Change $\int_0^4\int_0^{\sqrt{4x-x^2}} \sqrt{x^2+y^2}\,dydx$ to a polar double integral. Do not evaluate.

r

$$\begin{cases} 0\le x\le 4 \\ 0\le y\le \sqrt{4x-x^2}\end{cases}$$

$y^2 = 4x - x^2$

$x^2 - 4x + 4 + y^2 = 0 + 4$

$(x-2)^2 + y^2 = 4$

$C(2,0)$

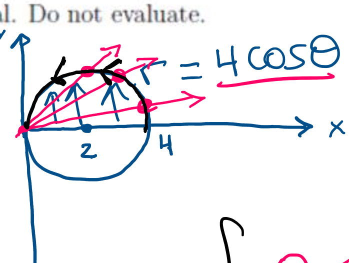
$r = 2$

$x^2 - 4x + y^2 = 0$

$\rightarrow r^2 - 4r\cos\theta = 0$

r

$r - 4\cos\theta = 0$



θ	$r=4\cos\theta$
0	4
$\frac{\pi}{6}$	$4\frac{\sqrt{3}}{2}$

$$\begin{cases} 0\le r\le 4\cos\theta \\ 0\le \theta\le \frac{\pi}{2}\end{cases}$$

$\pi/2 \quad 4\cos\theta$

$r = 2$

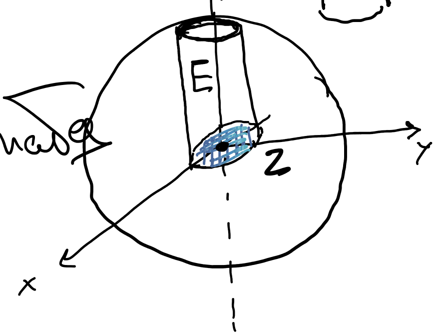
$$\int_0^{\pi/2}\int_0^{4\cos\theta} r \; r\,dr\,d\theta$$

$\frac{\pi}{6}$	$4\,\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$4\,\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$4\,\frac{1}{2}$
$\frac{\pi}{2}$	$\emptyset$

Problem 10. Set up but do not evaluate a double integral that gives the volume of the solid that lies above the xy-plane, below the sphere $x^2 + y^2 + z^2 = 81$ and inside the cylinder $x^2 + y^2 = 4$

$$Vol(E) = \iint_D f(x,y)\,dA$$

Polar Coordinates



$$f(x,y) = +\sqrt{81 - x^2 - y^2}$$

$$Vol(E) = \int_0^{2\pi}\int_0^{2} \sqrt{81 - r^2}\; r\,dr\,d\theta$$

#

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

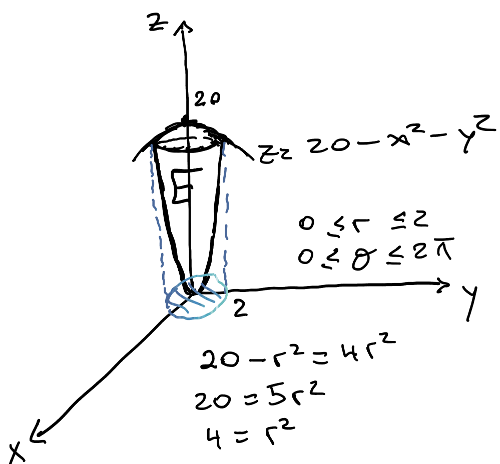
$x^2 + y^2 = r^2$

Problem 11. Find the volume of the solid bounded by the paraboloids $z = 20 - x^2 - y^2$ and $z = 4x^2 + 4y^2$.

$$Vol(E) = \iint_E \text{Top} - \text{Bottom}\; dA =$$

$$= \iint_E \left[(20 - x^2 - y^2) - (4x^2 + 4y^2)\right] dA$$

Polar Coord



$0 \le r \le 2$

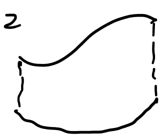
$0 \le \theta \le 2\pi$

$20 - r^2 = 4r^2$

$20 = 5r^2$

$4 = r^2$

$r = 2$



$$Vol(E) = \int_0^{2\pi}\int_0^{2} (20 - 5r^2)\, r\,dr\,d\theta$$

$$2\pi\left[\frac{\cancel{20}^{\,10}\, r^2}{\cancel{2}} - \frac{5}{4}r^4\right]_0^2 =$$

$$= 2\pi\left(40 - \frac{5}{\cancel{4}_1}\cdot\cancel{16}^{\,4} - \emptyset\right) = \boxed{40\pi}$$