Wir 3: Exam 1 Review

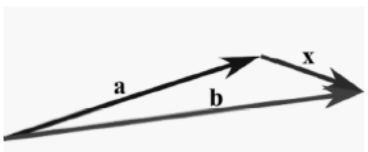
Sections 12.1-12.6 and 13.1-13.4

Problem 1. What is the equation of the sphere centered at $(6, 4, 12)$ with radius 6? Describe the intersection of this sphere with the three coordinate planes.

Problem 2. Let $\mathbf{a} = \langle 1, 2, -1\rangle$ and $\mathbf{b} = \langle 2, -1, 2\rangle$. Find the vector projection of $\mathbf{b}$ onto $\mathbf{a}$, that is $\text{proj}_{\mathbf{a}}\mathbf{b}$.

Problem 3. Let $\mathbf{a} = \langle -2, 2, 1\rangle$. Find a vector $\mathbf{b} = \langle b_1, b_2, b_3\rangle$ so that the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ equals -4, that is $\text{comp}_{\mathbf{a}}\mathbf{b} = -4$.

Problem 4. Use the figure below to answer the questions that follow.



a.) Write $\mathbf{x}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

b.) If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$, $|\mathbf{a}| = 7$, and $|\mathbf{b}| = 6$, find $\mathbf{a} \cdot \mathbf{b}$.

c.) If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$, $|\mathbf{a}| = 7$, and $|\mathbf{b}| = 6$, find $|\mathbf{a} \times \mathbf{b}|$ and determine whether $\mathbf{a} \times \mathbf{b}$ is directed into or out of the page.

Problem 5. Find a vector equation, a set of parametric equations, and symmetric equations for the line passing through the point $(-2, 3, 4)$ that is parallel to the vector $\langle 1, -4, 4\rangle$.

Problem 6. Consider the line that passes through the points $(4, 3, -1)$ and $(5, 3, 5)$. Where does this line intersect the three coordinate planes, and if it does not intersect one of the three coordinate planes, explain why not.

Problem 7. Find the equation of the plane that contains the point $(1, 2, -5)$ and is perpendicular to the vector $\langle -6, 4, -2\rangle$.

Problem 8. Find parametric equations for the line that passes through $(2, -1, 5)$ and is

a.) parallel to the line $\dfrac{x+1}{3} = \dfrac{y-6}{4} = z$.

b.) perpendicular to the plane $8x - 11y = 2z + 6$.

Problem 9. Consider the triangle with vertices $P(1, 0, 1)$, $Q(2, 3, 4)$ and $R(2, 1, 1)$.

a.) Find the angle at the vertex Q.

b.) Find the equation of the plane that passes through the points

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Problem 10. Find the equation of the plane that passes through the point $(1,0,1)$ and

a.) is perpendicular to the line $x = 9 - t$, $y = 7 + 2t$, $z = t$.

b.) contains line $x = 9 - t$, $y = 7 + 2t$, $z = t$.

Problem 11. Consider the plane P_1 given by the equation $2x - y + 3z = 7$ and the plane P_2 given by the equation $3x + y + 2z = 3$.

a.) Find the angle between the planes.

b.) Find a point, (x_0, y_0, z_0), that lies on both planes.

c.) Find a parametric equation for the line where the two planes intersect.

Problem 12. Consider the lines $\mathbf{r_1}(t) = \langle 1,2,0\rangle + t\,\langle 2,-2,2\rangle$ and $\mathbf{r_2}(v) = \langle 3,0,2\rangle + v\,\langle -2,2,0\rangle$.

a.) Find the point where the two lines intersect.

b.) Find an equation of the plane containing both of these lines.

Problem 13. Let $\mathbf{r}(t) = \left\langle t^2, \frac{t-1}{t^2-1}, \frac{\sin t}{t}\right\rangle$.

a.) Find the domain of $\mathbf{r}(t)$.

b.) Find $\lim\limits_{t\to 1} \mathbf{r}(\mathbf{t})$.

Problem 14. Let $\mathbf{r}(t) = \langle \cos(t^2), \sin(t^2), t^2\rangle$.

a.) Find the velocity and speed of the curve at time $t = \sqrt{\pi}$.

b.) Find $\mathbf{T}(\sqrt{\pi})$, the unit tangent vector, at $t = \sqrt{\pi}$.

c.) Find $\mathbf{a}(t)$, the acceleration vector, at time t.

d.) The length of the curve from the point $(1,0,0)$ to the point $(1,0,2\pi)$.

e.) The curvature of the curve traced out by $\mathbf{r}(t)$ when $t = \sqrt{\pi}$.

Problem 15. Find parametric equations for the tangent line to the curve $x = 4\sqrt{t}$, $y = t^2 - 10$, $z = \dfrac{4}{t}$ at $(8,6,1)$.

Problem 16. If $\mathbf{r}'(t) = \langle t, e^t, te^{3t}\rangle$ and $\mathbf{r}(0) = \langle 1,3,2\rangle$, find $\mathbf{r}(t)$.

Problem 17. Find $\displaystyle\int_0^1 \left(\frac{4t}{t^2+1}\mathbf{j} - \frac{1}{1+t^2}\mathbf{k}\right)\,dt.$

Problem 18. Given the curves $\mathbf{r}_1(t) = \langle 3t, t^2, t^3\rangle$ and $\mathbf{r}_2(v) = \langle \sin v, \sin(2v), 6v\rangle$ intersect at the origin, find the angle of intersection.

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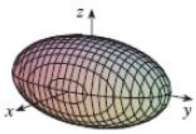
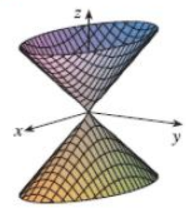
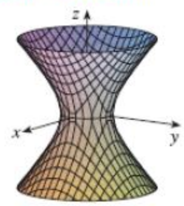
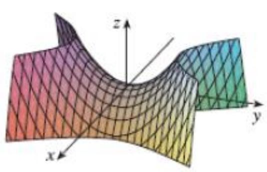
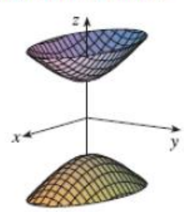
Problem 19. Be able to match an equation with the corresponding quadric surface.

Graphs of quadric surfaces

Surface	Equation	Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ All traces are ellipses. If $a=b=c$, the ellipsoid is a sphere.	Cone 	$\frac{z^2}{c^2}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x=k$ and $y=k$ are hyperbolas if $k\neq 0$ but are pairs of lines if $k=0$.
Elliptic Paraboloid 	$\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.	Hyperboloid of One Sheet 	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.
Hyperbolic Paraboloid 	$\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c<0$ is illustrated.	Hyperboloid of Two Sheets 	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Horizontal traces in $z=k$ are ellipses if $k>c$ or $k<-c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.

Identify the following quadric surfaces:

$$4x^2+9y^2-36z^2=36$$

$$16x^2+4y^2+4z^2-64x+8y+16z=0$$

$$-4x^2+y^2+16z^2-8x+10y+32z=0$$

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Problem 20. Match the parametric equations with the graphs (labeled I-VI)

a. $x = t\cos t,\ y = t,\ z = t\sin t,\ t \geq 0$

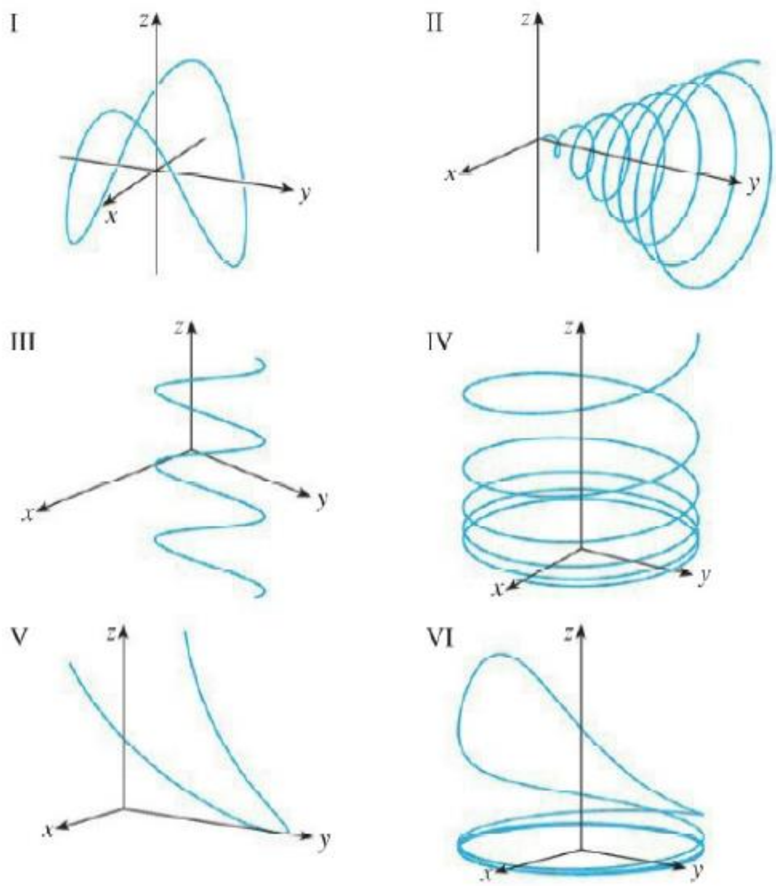
b. $x = \cos t,\ y = \sin t,\ z = \dfrac{1}{1+t^2}$

c. $x = t,\ y = \dfrac{1}{1+t^2},\ z = t^2$

d. $x = \cos t,\ y = \sin t,\ z = \cos(2t)$

e. $x = \cos 8t,\ y = \sin 8t,\ z = e^{0.8t}$

f. $x = \cos^2 t,\ y = \sin^2 t,\ z = t$



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