WIR 1

Wir 1: 12.1 to 12.3

SECTION 12.1 3d - coordinates

Problem 1. Find the center and radius of the sphere $x^2 + y^2 + z^2 + 4x - 2y - 8z = 5$. Does this sphere intersect the xz plane? If so, what is the intersection?

$$\underbrace{x^2 + 4x + 4} + \underbrace{y^2 - 2y + 1} + \underbrace{z^2 - 8z + 16} = 5 + 4 + 1 + 16$$

$$(x+2)^2 + (y-1)^2 + (z-4)^2 = 26$$

$c(-2, 1, 4)$ $\quad r = \sqrt{26}$

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xz plane $\boxed{y = 0}$

$$(x+2)^2 + \underbrace{(0-1)^2}_{+1} + (z-4)^2 = 26$$

$$\begin{cases} (x+2)^2 + (z-4)^2 = 25 \\ y = 0 \end{cases}$$

circle with center $(-2, 0, 4)$, $r = 5$ in the xz-plane

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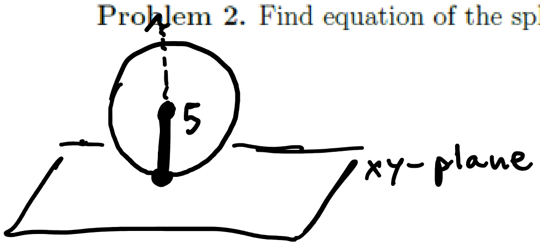
Problem 2. Find equation of the sphere with center $(1, 2, 5)$ that touches the xy plane.

$r = 5$



$$(x-1)^2 + (y-2)^2 + (z-5)^2 = 5^2$$

$$x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 10z + \cancel{25} = \cancel{25}$$

$$x^2 + y^2 + z^2 - 2x - 4y - 10z = -5$$

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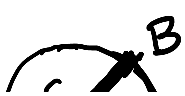
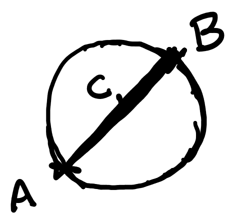
Problem 3. Find the equation of the sphere if one of their diameters has endpoints A$(5, 1, 5)$ and B$(7, 3, 9)$.

Midpoint of A and B $= \dfrac{A+B}{2}$



$c\left(\frac{12}{2}, \frac{4}{2}, \frac{14}{2}\right) = (6, 2, 7) \leftarrow$

Midpoint of $AB = \frac{A+B}{2}$



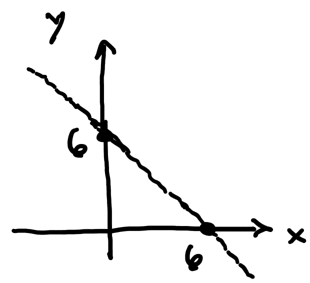
$C\left(\frac{12}{2}, \frac{4}{2}, \frac{14}{2}\right) = (6, 2, 7) \leftarrow$

$r = \frac{1}{2}|AB| = \frac{1}{2}\sqrt{(7-5)^2 + (3-1)^2 + (9-5)^2} =$

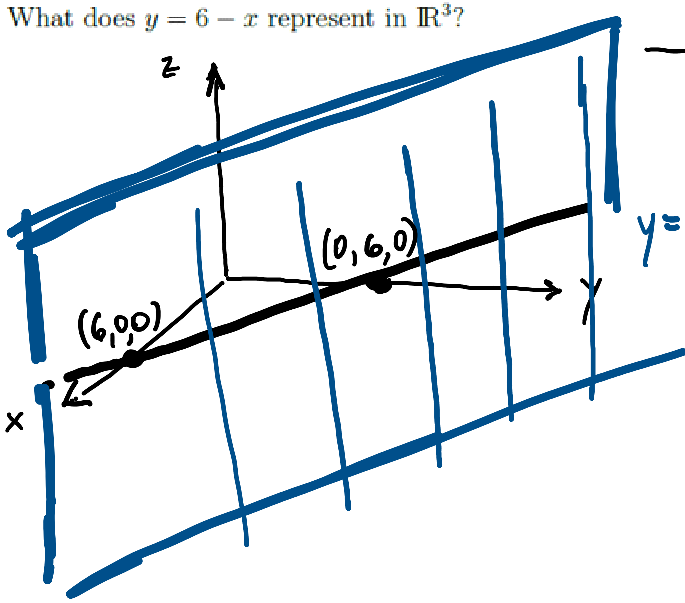
$= \frac{1}{2}\sqrt{4+4+16} = \frac{1}{2}\sqrt{24} = \frac{2}{2}\sqrt{6} = \sqrt{6} \leftarrow$

4.6

$(x-6)^2 + (y-2)^2 + (z-7)^2 = 6$ #



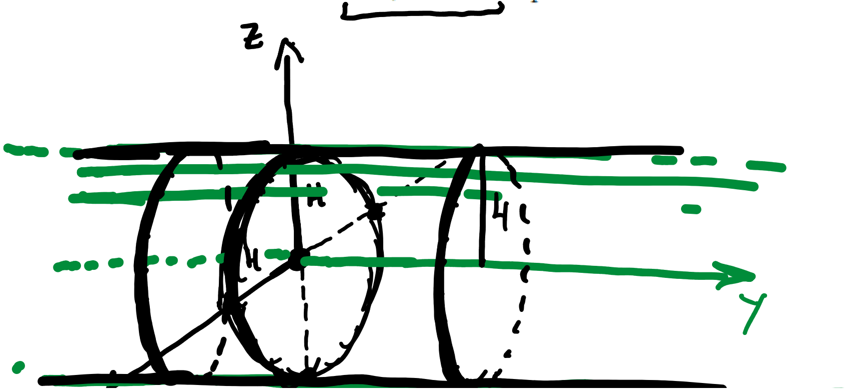
Problem 4. What does $y = 6 - x$ represent in $\mathbb{R}^3$?



$y = 6 - x$

plane

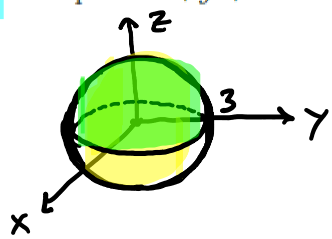
Problem 5. What does $\underbrace{x^2 + z^2 = 16}$ represent in $\mathbb{R}^3$?



Circular cylinder with axis the y-axis and radius 4.

Problem 6. Write a set of inequalities that describes the solid upper hemisphere $x^2+y^2+z^2=9$.

solid $\begin{cases} x^2+y^2+z^2 \leq 9 \\ z \geqslant 0 \end{cases}$



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SECTION 12.2 vectors

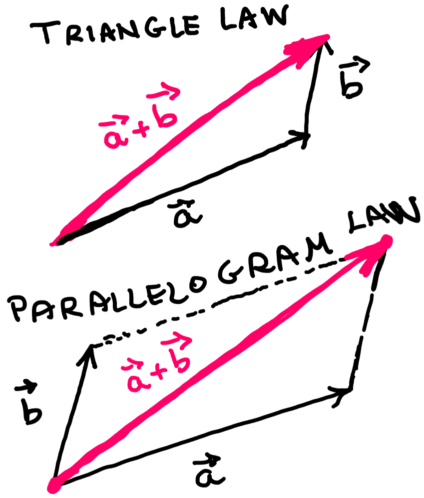
Problem 7. Give a graphical interpretation of vector sum and vector difference.

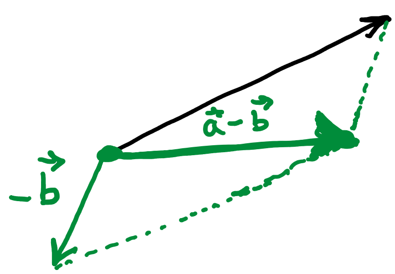
Vector Sum

vector difference

$\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$

TRIANGLE LAW





Problem 8. Given $\vec{a}=\langle -7,1,2\rangle$ and $\vec{b}=\langle 5,-1,1\rangle$, find a unit vector in the direction of $\underbrace{\vec{a}+2\vec{b}}$.

$2\vec{b}=2\langle 5,-1,1\rangle=\langle 10,-2,2\rangle$

Problem 8. Given $\vec{a} = \langle -7, 1, 2\rangle$ and $\vec{b} = \langle 5, -1, 1\rangle$, find a unit vector in the direction of $\underbrace{\vec{a}+2\vec{b}}$.

$2\vec{b} = 2\langle 5, -1, 1\rangle = \langle 10, -2, 2\rangle$

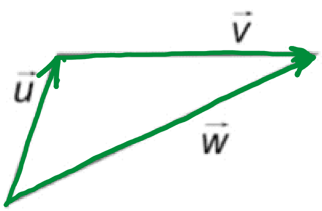
$\vec{a} + 2\vec{b} = \langle 3, -1, 4\rangle = \vec{v}$ $\qquad |\vec{v}| = \sqrt{9+1+16} = \sqrt{26}$

$\vec{u} = \frac{1}{|\vec{v}|}\vec{v} = \boxed{\left\langle \frac{3}{\sqrt{26}}, \frac{-1}{\sqrt{26}}, \frac{4}{\sqrt{26}} \right\rangle}$

* a vector 5 units long in direction opposite to $\vec{a}+2\vec{b}$

$\left\langle -\frac{15}{\sqrt{26}}, \frac{5}{\sqrt{26}}, -\frac{20}{\sqrt{26}} \right\rangle$

Problem 9. For the picture seen below, write **v** in terms of **u** and **w**.



$\vec{v} =$ $\vec{u} - \vec{w}$ / $\vec{w} - \vec{u}$

$\vec{v} = \vec{w} - \vec{u}$

$\vec{w} = \vec{u} + \vec{v} \rightarrow \vec{v} = \vec{w} - \vec{u}$.

Section 12.3 The dot product

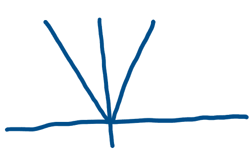
Problem 10. Compute $\mathbf{a} \cdot \mathbf{b}$ if

a.) $\vec{a} = \langle 4, 5, -1\rangle$ and $\vec{b} = \langle 2, 1, 3\rangle$. $\vec{a}\cdot\vec{b} = 8+5-3 = 10$ (acute angle)

b.) $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $\theta = 120°$. $\vec{a}\cdot\vec{b} = 2\cdot 5\cdot\cos 120° = 10\left(-\frac{1}{2}\right) = -5$ obtuse angle

c.) $|\vec{a}| = 6$, $|\vec{b}| = 4$ and $\underbrace{\vec{a}\text{ is perpendicular to }\vec{b}}$, $\vec{a}\cdot\vec{b} = 0$

d.) $|\vec{a}| = 6$, $|\vec{b}| = 4$ and $\underbrace{\vec{a}\text{ is parallel to }\vec{b}}_{\theta = 0}$, $\vec{a}\cdot\vec{b} = 6\cdot 4\cos 0 = 24$.



$\langle -2, 1, 3 \rangle \quad \langle 2, -1, -3 \rangle$

Problem 11. Are the vectors $-8\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}$ and $6\mathbf{i} - 3\mathbf{j} - 9\mathbf{k}$ parallel, perpendicular, or neither?

parallel = scalar multiples of each other

$$\underbrace{-8\left(\frac{6}{-8}\right)}_{-\frac{3}{4}} = 6 \qquad -\frac{3}{4}\langle -8, 4, 12 \rangle = \langle 6, -3, -9 \rangle$$

The vectors are parallel.

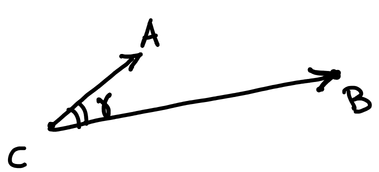
perpendicular? NO

DOT PRODUCT

$-48 - 12 - 108 \neq 0$

not perpendicular

Problem 12. The points $A(0, -1, 6)$, $B(2, 1, -3)$ and $C(5, 4, 2)$ form a triangle. Find $\angle C$.



$$\cos\gamma = \frac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}||\vec{CB}|} = \frac{10}{\sqrt{66}\sqrt{43}}$$

$$\gamma = \cos^{-1}\left(\frac{10}{\sqrt{66}\sqrt{43}}\right)$$

$\vec{CA} = A - C = \langle -5, -5, 4 \rangle$

$\vec{CB} = B - C = \langle -3, -3, -5 \rangle$

$\vec{CA} \cdot \vec{CB} = 15 + 15 - 20 = 10$

$|\vec{CA}| = \sqrt{25 + 25 + 16} = \sqrt{66}$

$|\vec{CB}| = \sqrt{9 + 9 + 25} = \sqrt{43}$

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$18 + 25 = 43$

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$

Problem 13. Find the vector and scalar projection of $\langle 1, 2, 5 \rangle$ ($\vec{a}$) onto $\langle 0, 7, 4 \rangle$ ($\vec{b}$).

$$\text{comp}_{\vec{b}}\,\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$\text{proj}_{\vec{b}}\,\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \cdot \frac{\vec{b}}{|\vec{b}|} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\right)\vec{b}$$

$\vec{a} \cdot \vec{b} = 0 + 14 + 20 = 34$

$|\vec{b}| = \sqrt{0 + 49 + 16} = \sqrt{65}$

$$\begin{array}{r} 49 \\ 16 \\ \hline 65 \end{array} \qquad 65/5 = 13$$

$$\operatorname{comp}_{\vec{b}} \vec{a} = \frac{34}{\sqrt{65}}$$

$$\operatorname{proj}_{\vec{b}} \vec{a} = \frac{34}{65} \langle 0, 7, 4 \rangle = \left\langle 0, \frac{238}{65}, \frac{136}{65} \right\rangle$$

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