WIR 7

Wir 7: Sections 15.1, 15.2, 15.3

Section 15.1

Problem 1. Find $\int_0^{\pi/4} \underline{x\sin(3y)\,dy} = -\frac{1}{3}x\left[\cos(3y)\right]_0^{\pi/4} =$

$$= -\frac{1}{3}x\left[\underbrace{\cos\frac{3\pi}{4}}_{-1} - \underbrace{\cos 0}_{1}\right] = \boxed{\frac{2}{3}x}$$

Problem 2. Find $\int_1^e \left[\frac{y\ln(x)}{x}\,dx\right]$

$\boxed{u = \ln x}$ $\quad du = \frac{1}{x}dx$

$$y\int_{\ln 1 = 0}^{\ln e = 1} u\,du = y\,\frac{u^2}{2}\Big|_0^1 = \frac{y}{2}(1-0) = \boxed{\frac{y}{2}}$$

Problem 3. Evaluate $\underline{\int_0^2\int_0^3 (xy+x+y)\,dy\,dx}$ and $\underline{\int_0^3\int_0^2 (xy+x+y)\,dx\,dy}$

$$\left[\frac{1}{2}xy^2 + xy + \frac{1}{2}y^2\right]_{y=0}^{y=3}$$

$$\int^2 \left[\frac{9}{2}x + 3x + \frac{9}{2} - 0\right]dx$$

$$\left[\frac{x^2}{2}y + \frac{x^2}{2} + xy\right]_{x=0}^{x=2}$$

$$\int^3 \left[2y + 2 + 2y - 0\right]dy =$$

$$\int_0^2 \left[\frac{9}{2}x + 3x + \frac{9}{2} - 0\right] dx$$

$$\frac{9}{2}\frac{x^2}{2} + \frac{3}{2}x^2 + \frac{9}{2}x \Big|_0^2 =$$

$$= 9 + 6 + 9 - 0 =$$

$$= 24$$

$$\int_0^3 [2y + 2 + 2y - 0]\, dy =$$

$$= y^2 + 2y + y^2 \Big]_0^3 =$$

$$= 9 + 6 + 9 =$$

$$= 24$$

Fubini's Theorem: If f is continuous on the rectangle $R = [a, b] \times [c, d]$, then

(1) $\underbrace{\iint_R f(x,y)\, dA}_{\text{double integral}} = \underbrace{\int_a^b \int_c^d f(x,y)\, dydx = \int_c^d \int_a^b f(x,y)\, dxdy}_{\text{Iterated Integrals}}.$

(2) In the case where $f(x,y) = g(x)h(y)$, then

$$\iint_R f(x,y)\, dA = \int_a^b \int_c^d g(x)h(y)\, dydx = \int_a^b g(x)\, dx \int_c^d h(y)\, dy$$

Problem 4. Find $\iint_R \frac{x}{y^2}\, dA$, where $R = \underbrace{[0,4]}_{x} \times \underbrace{[1,2]}_{y}$

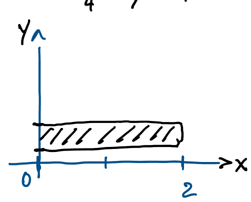
$$\int_0^4 x\, dx \cdot \int_1^2 \frac{1}{y^2}\, dy = \left[\frac{x^2}{2}\right]_0^4 \left[-\frac{1}{y}\right]_1^2 =$$

$$= \left(\frac{16}{2} - \frac{0}{2}\right)\Big(\underbrace{-\frac{1}{2} - -\frac{1}{1}}_{-\frac{1}{2}+1}\Big) = 8 \cdot \frac{1}{2} = 4$$

Problem 5. Find $\iint_R x \sec^2 y\, dA$, where $R = \{(x,y) | 0 \le x \le 2, \cancel{1 \le y \le \frac{\pi}{4}}\}$ TYPO

$\frac{\pi}{4} \le y \le 1$

$$\int_0^2 \int_{\frac{\pi}{4}}^1 x \sec^2 y\, dy\, dx = \left(\int_0^2 x dx\right)\left(\int_{\frac{\pi}{4}}^1 \sec^2 y\, dy\right) =$$



$$= \left[\frac{x^2}{2}\right]_0^2 \left[\tan y\right]_{\frac{\pi}{4}}^1 =$$

$$= \left(\frac{4}{2} - 0\right)\left(\tan 1 - \underbrace{1}_{\tan\frac{\pi}{4}}\right) = 2\left((\tan 1) - 1\right) \text{ POSITIVE}$$

$= [2]_0 \, [\tan y]_{\frac{\pi}{4}} \quad \tan\frac{\pi}{4}$

$= \left(\frac{4}{2} - 0\right)\left(\tan 1 - 1\right) = 2\left((\tan 1) - 1\right)$ POSITIVE

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Problem 6. Find $\iint_R e^{2x+y}\, dA$, where $R = \underbrace{[0, \ln 2]}_{x} \times \underbrace{[0, \ln 3]}_{y}$

$$\int_0^{\ln 3}\left\{\int_0^{\ln 2} e^{2x} \cdot e^{y}\, dx\right\} dy = \int_0^{\ln 3} e^{y}\, dy \int_0^{\ln 2} e^{2x}\, dx$$

$$\left[e^{y}\right]_0^{\ln 3} \left[\frac{1}{2} e^{2x}\right]_0^{\ln 2} =$$

$e^{\ln 3}$

$$= (3 - 1) \; \frac{1}{2}\left[e^{2\ln 2} - e^{0}\right] =$$

$\hookrightarrow e^{0}$

$$= 2 \cdot \frac{1}{2}\left[4 - 1\right] = \boxed{3}$$

Problem 7. Find $\iint_R (y\cos(xy))\, dA$, where $R = \overbrace{[0, 2]}^{x} \times \overbrace{[0, \pi]}^{y}$

$$\int_0^{\pi}\left\{\int_0^{2} y \cos(xy)\, dx\right\} dy$$

$$\left[\cancel{y}\frac{1}{\cancel{y}} \sin(xy)\right]_{x=0}^{x=2} =$$

$= \sin(2y) - \cancel{\sin 0}$

$$\int_0^{\pi} \sin(2y)\, dy =$$

$$= \left(-\frac{1}{2}\cos(2y)\right)\Bigg|_{y=0}^{\pi} = -\frac{1}{2}\left[1 - 1\right] = \phi$$

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$\int\left\{\int y\cos(xy)\, dy\right\} dx$ TABULAR INTEGRATION

by parts

Problem 8. Find the volume of the solid S that is bounded by the paraboloid $x^2 + y^2 + z = 16$, $z = 0$, $0 \le x \le 4$, $0 \le y \le 4$.

$z = 16 - x^2 - y^2$

Problem 8. Find the volume of the solid S that is bounded by the paraboloid $x^2+y^2+z=16$, $z=0$, $0 \le x \le 4$, $0 \le y \le 4$.

$z = 16 - x^2 - y^2$

$$\mathrm{Vol}(E) = \iint_R (16 \underbrace{- x^2 - y^2}_{-(x^2+y^2)})\, dA \overset{\text{Polar coord}}{=}$$

$$\int_0^{\pi/2} \int_0^4 \underbrace{(16 - r^2)\, r}_{16r - r^3}\, dr\, d\theta =$$

$$\left[8r^2 - \frac{r^4}{4}\right]_0^4 = $$

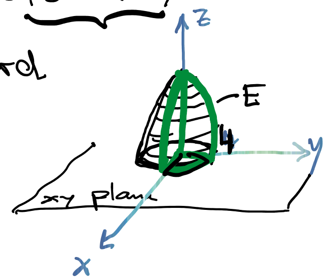
$$= 8 \cdot 16 - \cancel{4} \cdot \frac{4^3}{\cancel{4}} - 0 =$$

$$= 128 - 64 = 64$$

64

$\theta \Big|_0^{\pi/2} = \frac{\pi}{2}$

Answer: $\frac{64\pi}{2} = \boxed{32\pi}$

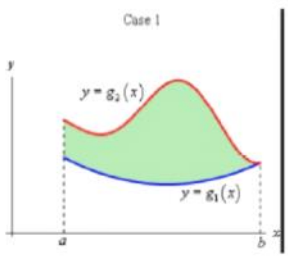


$\begin{cases} 0 \le r \le 4 \\ 0 \le \theta \le \frac{\pi}{2} \end{cases}$

$\frac{16}{4}$

Section 15.2

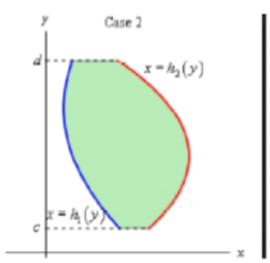
Type I: A plane region D is said to be of type I if it lies between the graphs of two continuous functions of x, that is $D = \{(x,y) | a \le x \le b, g_1(x) \le y \le g_2(x)\}$.



If f is continuous on a type I region $D = \{(x,y) | a \le x \le b, g_1(x) \le y \le g_2(x)\}$, then

$$\iint_R f(x,y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy dx$$

Type II: A plane region D is said to be of type II if it lies between the graphs of two continuous functions of y, that is $D = \{(x,y) | c \le y \le d, h_1(y) \le x \le h_2(y)\}$.



If f is continuous on a type II region $D = \{(x,y) | c \le y \le d, h_1(y) \le x \le h_2(y)\}$, then

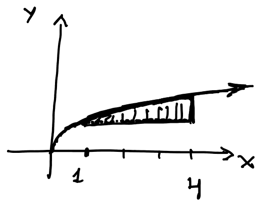
$$\iint_D f(x,y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dx dy$$

Problem 9. Evaluate $\int_1^4 \int_1^{\sqrt{x}} (x+y)\, dy dx$

$\begin{cases} 1 \le y \le \sqrt{x} \\ 1 \le x \le 4 \end{cases}$

Problem 9. Evaluate $\int_1^4 \left\{ \int_1^{\sqrt{x}} (x+y)\, dy \right\} dx$

$$\begin{cases} 1 \le y \le \sqrt{x} \\ 1 \le x \le 4 \end{cases}$$



$$\left[xy + \frac{1}{2} y^2 \right]_{y=1}^{y=\sqrt{x}} =$$

$$= \underbrace{x\sqrt{x}}_{x^{1+\frac{1}{2}}} + \underbrace{\frac{x}{2} - \left(x + \frac{1}{2}\right)}_{\frac{x}{2} - x} =$$

$$= \int_1^4 x^{3/2} - \frac{x}{2} - \frac{1}{2} \; dx$$

$$4^{5/2} = \sqrt{4}^{\,5} = 2^5$$

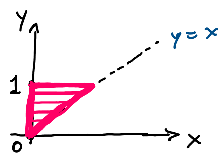
$$\left[\frac{2}{5} x^{5/2} - \frac{x^2}{4} - \frac{1}{2} x \right]_1^4 = \frac{2}{5} \cdot 32 - 4 - 2 - \left(\frac{2}{5} - \frac{1}{4} - \frac{1}{2} \right) =$$

$$= \frac{64}{5} - 6 - \frac{2}{5} + \frac{1}{4} + \frac{1}{2} =$$

$$= \frac{256 - 120 - 8 + 5 + 10}{20} = \frac{136+7}{20} = \boxed{\frac{143}{20}}$$

Problem 10. Evaluate $\int_0^1 \int_0^y (3 + x^2 y)\, dx dy$

$$0 \le x \le y$$
$$0 \le y \le 1$$



$$\left[3x + \frac{1}{3} x^3 y \right]_{x=0}^{x=y} = 3y + \frac{1}{3} y^4 - 0$$

$$\int_0^1 \left(3y + \frac{1}{3} y^4 \right) dy = \frac{3}{2} y^2 + \frac{1}{3} \frac{1}{5} y^5 \Big|_0^1 =$$

$$= \frac{3}{2} + \frac{1}{15} - 0 = \frac{45+2}{30} = \frac{47}{30}$$

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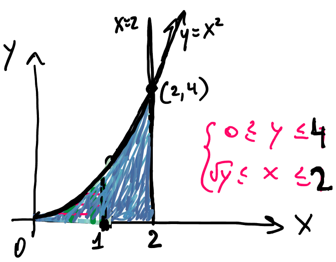
Problem 11. Sketch the region of integration and evaluate $\iint_D x e^y \, dA$ where D is the region bounded by $y = 0$, $y = x^2$ and $x = 2$



$$\begin{cases} 0 \le y \le 4 \\ \sqrt{y} \le x \le 2 \end{cases}$$

$$\begin{cases} 0 \le x \le 2 \end{cases}$$

$$\int_0^2 \left\{ \int_0^{x^2} x e^y \, dy \right\} dx$$

$$\left[x e^y \right]_{y=0}^{x^2} =$$

$$= x e^{x^2} - x \cdot 1$$

$$\int_0^2 \left(x e^{x^2} - x \right) dx =$$

$u = x^2 \quad du = 2x\, dx \ldots$

$$\begin{cases} 0 \le x \le 2 \\ 0 \le y \le x^2 \end{cases}$$

$$\int_0 (\underbrace{xe^{x^2}}_{u=x^2\ \ du=2x\,dx\ldots} - x)\,dx =$$

$$= \left[\frac{1}{2}e^{x^2} - \frac{x^2}{2}\right]_{x=0}^{x=2} =$$

$$= \frac{1}{2}e^4 - \frac{4}{2} - \left(\frac{1}{2}e^0 - 0\right) =$$

$$= \frac{e^4}{2} - 2 - \frac{1}{2} = \frac{e^4}{2} - \frac{5}{2} = \frac{e^4-5}{2}$$

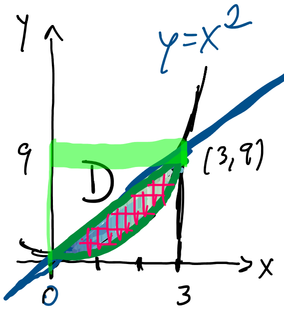
Problem 12. Set up but do not evaluate both a type I and type II integral for $\iint_D f(x,y)\,dA$, where D is the region bounded by $y = x^2$ and $y = 3x$.

$(0,0)$

$x^2 = 3x$

$x^2 - 3x = 0$

$x(x-3) = 0$



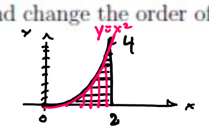
$$\begin{cases} 0 \le x \le 3 \\ x^2 \le y \le 3x \end{cases} \qquad \begin{cases} 0 \le y \le 9 \\ \frac{y}{3} \le x \le \sqrt{y} \end{cases}$$

$$\int_0^3 \int_{x^2}^{3x} f(x,y)\,dy\,dx = \int_0^9 \int_{\frac{y}{3}}^{\sqrt{y}} f(x,y)\,dx\,dy$$

Problem 13. Sketch the region of integration and change the order of integration.

(i) $\int_0^4 \int_{\sqrt{y}}^2 f(x,y)\,dx\,dy$

$$\begin{cases} 0 \le y \le 4 \\ \sqrt{y} \le x \le 2 \end{cases}$$

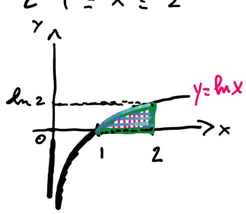


$$\begin{cases} 0 \le x \le 2 \\ 0 \le y \le x^2 \end{cases}$$

$$(i) = \int_0^2 \int_0^{x^2} f(x,y)\,dy\,dx$$

(ii) $\int_1^2 \int_0^{\ln x} f(x,y)\,dy\,dx$

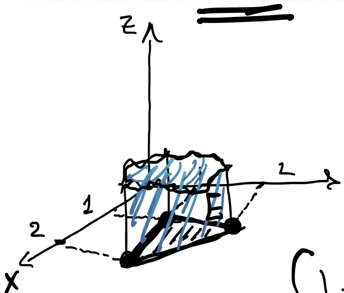
$$\begin{cases} 0 \le y \le \ln x \\ 1 \le x \le 2 \end{cases}$$



$$\begin{cases} 0 \le y \le \ln 2 \\ e^y \le x \le 2 \end{cases}$$

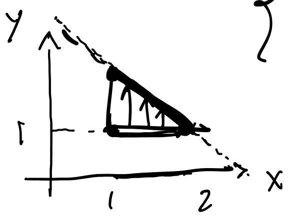
$$(ii) = \int_0^{\ln 2} \int_{e^y}^{2} f(x,y)\,dx\,dy$$

Problem 14. Set up but do not evaluate a double itegral that gives the volume of the solid under the surface $z = xy$ and above the triangle with vertices $(1,1)$, $(1,2)$ and $(2,1)$. T



$$Vol(E) = \iint_T xy\,dA = \int_1^2 \left\{ \int_1^{3-x} xy\,dy \right\} dx$$

$$\begin{cases} 1 \le x \le 2 \\ 1 \le y \le -x+3 \end{cases}$$



$y = -x + 3$

$$x\frac{1}{2}\left[y^2\right]_{y=1}^{y=3-x} =$$

$$= \frac{x}{2}\left[9 - 6x + x^2 - 1\right] =$$

$(3-x)^2$

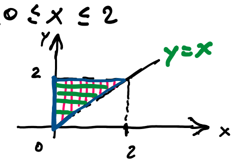
$$\int_1^2 4x - 3x^2 + \frac{x^3}{2}\,dx =$$

$$\left[2x^2 - x^3 + \frac{1}{2}\frac{x^4}{4}\right]_1^2 =$$

$$= \cancel{8} - \cancel{8} + \cancel{2} - \cancel{2} + 1 - \frac{1}{8} = +1 - \frac{1}{8} = \frac{7}{8}$$

Problem 15. Evaluate $\int_0^2 \int_x^2 e^{-y^2}\,dy\,dx = \int_0^2 \left\{ \int_0^y e^{-y^2} dx \right\} dy$

$$\begin{cases} x \le y \le 2 \\ 0 \le x \le 2 \end{cases}$$



$$\begin{cases} 0 \le y \le 2 \\ 0 \le x \le y \end{cases}$$

$$e^{-y^2}[x]_0^y =$$

$$= e^{-y^2}(y - 0) = ye^{-y^2}$$

$$\int_0^2 ye^{-y^2}dy \qquad \boxed{u = -y^2} \quad du = -2y\,dy$$

$$\int_0^{-4} -e^u \frac{du}{2} = +\int_{-4}^0 \frac{1}{2}e^u\,du =$$

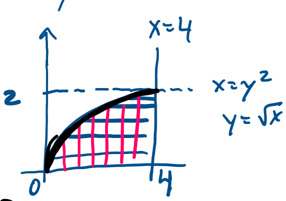
$$= \frac{1}{2}e^u\Big|_{-4}^0 = \frac{1}{2}\left(e^0 - e^{-4}\right) =$$

$$= \boxed{\frac{1}{2}\left(1 - \frac{1}{e^4}\right)}$$

Problem 16. Evaluate $\int_0^2 \int_{y^2}^4 \sqrt{x}\sin x\,dx\,dy = \int_0^4 \left\{ \int_0^{\sqrt{x}} \sqrt{x}\sin x\,dy \right\} dx$

$y^2 \le x \le 4$

$0 \le y \le 2$



$$\begin{cases} 0 \le x \le 4 \\ 0 \le y \le \sqrt{x} \end{cases}$$

$$\sqrt{x}\sin x\left[y\right]_0^{\sqrt{x}} =$$

$$= \sqrt{x}(\sin x)(\sqrt{x} - 0) = x\sin x$$

$$\int x\sin x\,dx = \left[-x\cos x + \sin x\right]_0^4 =$$

$$\begin{cases} 0 \le x \le 4 \\ 0 \le y \le \sqrt{x} \end{cases}$$

$\int x \sin x \, dx = \left[-x \cos x + \sin x \right]_0$

$= \boxed{-4\cos 4 + \sin 4} - 0 \cancel{+ \sin 0}$

DERIV	INTEGR
x	$\sin x$
1	$-\cos x$
0	$-\sin x$

Section 15.3

Recall: If $P(x, y)$ is a point in the xy-plane, we can represent the point P in polar form: Let r be the distance from O to P and let θ be the angle between the polar axis and the line OP. Then the point P is represented by the ordered pair (r, θ), and r, θ are called the **polar coordinates** of P.

Connecting polar coordinates with rectangular coordinates:

a.) $x = r\cos(\theta)$, $y = r\sin(\theta)$

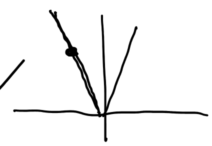
b.) $\tan(\theta) = \dfrac{y}{x}$, thus $\theta = \arctan\left(\dfrac{y}{x}\right)$.

c.) $x^2 + y^2 = r^2$

Problem 1. Find the cartesian coordinates of the polar point $\left(2, \dfrac{2\pi}{3}\right)$.

$$\begin{cases} x = 2\cos\frac{2\pi}{3} = 2\left(-\frac{1}{2}\right) = -1 \\ y = 2\sin\frac{2\pi}{3} = 2\,\frac{\sqrt{3}}{2} = \sqrt{3} \end{cases}$$

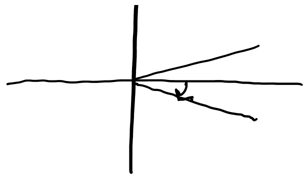
$(-1, \sqrt{3})$ ✓



Problem 2. Find the polar coordinates of the rectangular point $(\sqrt{3}, -1)$.

$$\begin{cases} r = \sqrt{x^2 + y^2} = \sqrt{3 + 1} = 2 \\ \theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6} \end{cases}$$

$\left(2, -\frac{\pi}{6}\right)$



Problem 3. Find a cartesian equation for the curve described by $r = 2\sin\theta$.

$$r = 2\sin\theta$$

$$y = r\sin\theta$$

$$(r)\; r = 2\frac{y}{\cancel{r}}\; \cancel{(r)}$$

$$\sin\theta = \frac{y}{r}$$

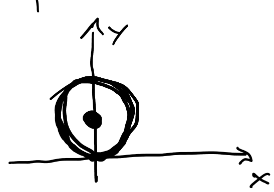
$$r^2 = 2y$$

$$x^2 + y^2 = 2y \rightarrow x^2 + y^2 - 2y = 0$$

$$x^2 + (y^2 - 2y + 1) - 1 = 0$$

$$x^2 + (y-1)^2 = 1$$

$C(0,1)$ $\quad r = 1$



Problem 4. Find a polar equation for $y = 1 + 3x$

$$\cancel{r\sin\theta} = 1 + 3r\cos\theta - r\sin\theta$$

$$-1 = r(3\cos\theta - \sin\theta)$$

$$r = \frac{-1}{3\cos\theta - \sin\theta}$$

Change to Polar Coordinates in a Double Integral: If f is continuous on a polar rectangle R given by $0 \le a \le r \le b$, $\alpha \le \theta \le \beta$, where $0 \le \beta - \alpha \le 2\pi$, then

$$\iint f(x,y)\,dA = \int^{\beta} \int^{b} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

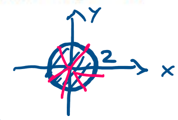
R given by $0 \le a \le r \le b$, $\alpha \le \theta \le \beta$, where $0 \le \beta - \alpha \le 2\pi$, then

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

Problem 5. Evaluate $\iint_R (x+2)\,dA$, where R is the region bounded by the circle $x^2+y^2=4$.

$$\int_0^{2\pi}\int_0^2 (r\cos\theta + 2)\,r\,dr\,d\theta =$$

$r^2\cos\theta + 2r$



$$\begin{cases} 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \end{cases}$$

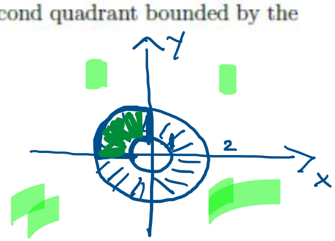
$$\left[\frac{r^3}{3}\cos\theta + r^2\right]_{r=0}^{r=2}$$

$$\int_0^{2\pi} \frac{8}{3}\cos\theta + 4 - 0 \quad d\theta$$

$$\left[\frac{8}{3}\sin\theta + 4\theta\right]_0^{2\pi} = 4\cdot 2\pi - 0 = \boxed{8\pi}$$

Problem 6. Evaluate $\iint_R 4y\,dA$, where R is the region in the second quadrant bounded by the circles $x^2+y^2=1$ and $x^2+y^2=4$.

$$\int_{\pi/2}^{\pi}\left\{\int_1^2 4r\sin\theta\, r\,dr\right\}d\theta$$



$$\begin{cases} 1 \le r \le 2 \\ \frac{\pi}{2} \le \theta \le \pi \end{cases}$$

$$\left[\frac{4}{3}r^3\sin\theta\right]_{r=1}^{2} =$$

$$= \frac{4}{3}\cdot 8\sin\theta - \frac{4}{3}\sin\theta$$

$$= \frac{28}{3}\sin\theta$$

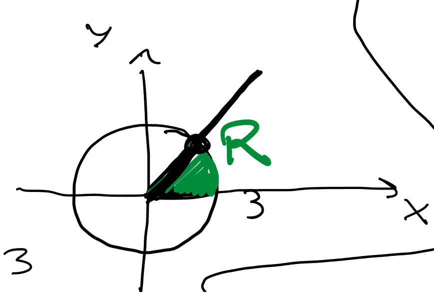
$$\int_{\pi/2}^{\pi} \frac{28}{3}\sin\theta\,d\theta =$$

$$= -\frac{28}{3}\left[\cos\theta\right]_{\frac{\pi}{2}}^{\pi} = -\frac{28}{3}(-1-0) = \boxed{\frac{28}{3}}$$

Problem 7. Evaluate $\iint_R 3x^2\,dA$, where R is the region in the first quadrant enclosed by the by the circle $x^2+y^2=9$ and the lines $y=0$ and $y=x$.

$$\rightarrow \frac{3}{4}\cdot 81\cos^2\theta - 0$$

$$\int_0^{\pi/4}\left\{\int_0^3 3r^2\cos^2\theta\, r\,dr\right\}d\theta$$



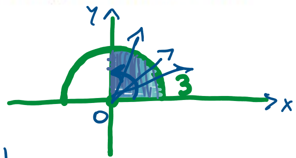
$\pi/4 \quad \Big[\quad \Big]^3$

$$\frac{243}{4}\int_0^{\pi/4}\cos^2\theta\, d\theta \qquad \left[\frac{3}{4}r^4\cos^2\theta\right]_0^3 = \qquad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$\frac{243}{4}\cdot\frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/4} = \frac{243}{8}\left[\frac{\pi}{4}+\frac{1}{2} - 0\right] = \frac{243}{8}\left(\frac{\pi}{4}+\frac{1}{2}\right) \checkmark$$

Problem 8. Change $\int_0^3\int_0^{\sqrt{9-x^2}} x^2\, dydx$ to a polar double integral. Do not evaluate.

$$\boxed{\int_0^{\pi/2}\int_0^3 r^2\cos^2\theta\, r\, dr\, d\theta}$$

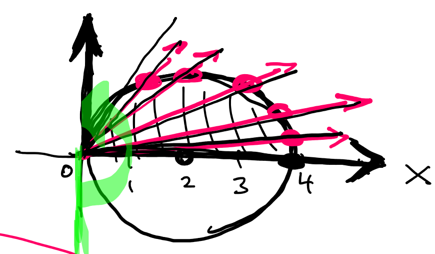


$0 \le r \le 3$

$0 \le \theta \le \frac{\pi}{2}$

$$\boxed{\int_0^{\frac{\pi}{2}}\int_0^{4\cos\theta} r\, r\, dr\, d\theta}$$

Problem 9. Change $\int_0^4\int_0^{\sqrt{4x-x^2}} \boxed{\sqrt{x^2+y^2}}\, dydx$ to a polar double integral. Do not evaluate.



$0 \le r \le 4\cos\theta$

$0 \le \theta \le \frac{\pi}{2}$

$y = \sqrt{4x - x^2}$

$y^2 = 4x - x^2$

$\boxed{x^2 + y^2 - 4x = 0} \to r^2 - 4r\cos\theta = 0 \to r(r - 4\cos\theta) = 0$

$r = 4\cos\theta$

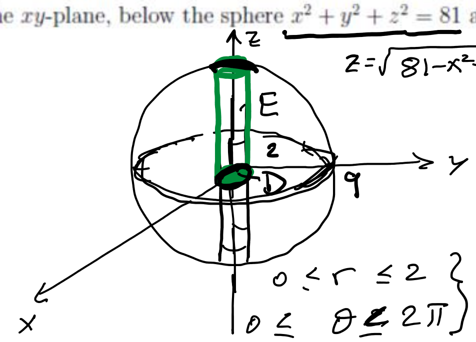
$\underbrace{x^2 - 4x + 4}_{} + y^2 = 4 + 0$

$(x-2)^2 + y^2 = 4 \qquad c(2,0)$

$r = 2$

Problem 10. Set up but do not evaluate a double integral that gives the volume of the solid that lies above the xy-plane, below the sphere $x^2 + y^2 + z^2 = 81$ and inside the cylinder $x^2 + y^2 = 4$

$z = \sqrt{81 - x^2 - y^2}$



$0 \le r \le 2$
$0 \le \theta \le 2\pi$

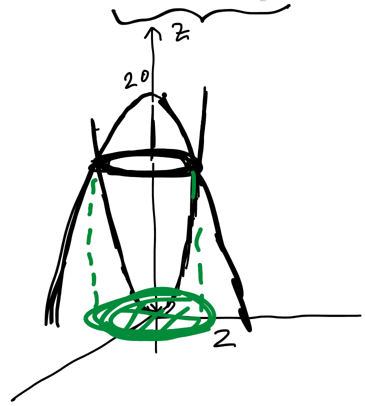
$$Vol(E) = \iint_D \sqrt{81 - x^2 - y^2}\, dA =$$

$$= \int_0^{2\pi} \int_0^2 \sqrt{81 - r^2}\; r\, dr\, d\theta$$

Top - Bottom

Problem 11. Find the volume of the solid bounded by the paraboloids $z = 20 - x^2 - y^2$ and $z = 4x^2 + 4y^2$.

$20 - r^2 = 4r^2$
$20 = 5r^2 \rightarrow r^2 = 4$
$r = 2$



$$\int_0^{2\pi} \int_0^2 \left[(20 - r^2) - (4r^2)\right] r\, dr\, d\theta$$

$(20 - 5r^2)r$

$20r - 5r^3$

$\frac{\cancel{20}^{10} r^2}{\cancel{2}} - \frac{5}{4} r^4 \Big|_0^2$

$10 \cdot 4 - \frac{5}{\cancel{4}} \cdot \cancel{16}^{4} - 0$

$40 - 20 = \underline{20}$

$\theta \Big|_0^{2\pi} = 2\pi$

Answer = $\underline{40\pi}$