Wir 7: Sections 15.1, 15.2, 15.3

Section 15.1

Problem 1. Find $\int_0^{\pi/4} x\sin(3y)\,dy$

Problem 2. Find $\int_1^e \frac{y\ln(x)}{x}\,dx$

Problem 3. Evaluate $\int_0^2\int_0^3 (xy+x+y)\,dydx$ and $\int_0^3\int_0^2 (xy+x+y)\,dxdy$

Fubini's Theorem: If f is continuous on the rectangle $R=[a,b]\times[c,d]$, then

(1) $\iint_R f(x,y)\,dA = \int_a^b\int_c^d f(x,y)\,dydx = \int_c^d\int_a^b f(x,y)\,dxdy.$

(2) In the case where $f(x,y)=g(x)h(y)$, then

$$\iint_R f(x,y)\,dA = \int_a^b\int_c^d g(x)h(y)\,dydx = \int_a^b g(x)\,dx\int_c^d h(y)\,dy$$

Problem 4. Find $\iint_R \frac{x}{y^2}\,dA$, where $R=[0,4]\times[1,2]$

Problem 5. Find $\iint_R x\sec^2 y\,dA$, where $R=\{(x,y)|0\le x\le 2, 1\le y\le \frac{\pi}{4}\}$

Problem 6. Find $\iint_R e^{2x+y}\,dA$, where $R=[0,\ln 2]\times[0,\ln 3]$

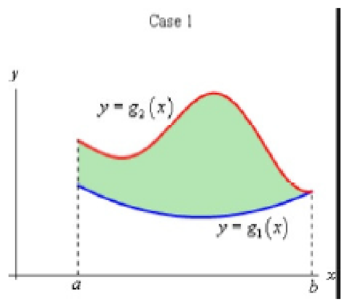
Problem 7. Find $\iint_R (y\cos(xy))\,dA$, where $R=[0,2]\times[0,\pi]$

Problem 8. Find the volume of the solid S that is bounded by the paraboloid $x^2+y^2+z=16$, $z=0$, $0\le x\le 4$, $0\le y\le 4$.

With thanks to Amy Austin for generously sharing all of her WIR problems from last semester.

Section 15.2

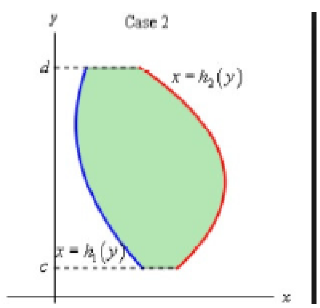
Type I: A plane region D is said to be of type I if it lies between the graphs of two continuous functions of x, that is $D = \{(x, y)|a \leq x \leq b, g_1(x) \leq y \leq g_2(x)\}$.



If f is continuous on a type I region $D = \{(x, y)|a \leq x \leq b, g_1(x) \leq y \leq g_2(x)\}$, then

$$\iint_R f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dydx$$

Type II: A plane region D is said to be of type II if it lies between the graphs of two continuous functions of y, that is $D = \{(x, y)|c \leq y \leq d, h_1(y) \leq x \leq h_2(y)\}$.



If f is continuous on a type II region $D = \{(x, y)|c \leq y \leq d, h_1(y) \leq x \leq h_2(y)\}$, then

$$\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dxdy$$

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Problem 9. Evaluate $\int_1^4 \int_1^{\sqrt{x}} (x+y)\, dydx$

Problem 10. Evaluate $\int_0^1 \int_0^y (3 + x^2 y)\, dxdy$

Problem 11. Sketch the region of integration and evaluate $\iint_D xe^y\, dA$ where D is the region bounded by $y = 0$, $y = x^2$ and $x = 2$

Problem 12. Set up but do not evaluate both a type I and type II integral for $\iint_D f(x, y)\, dA$, where D is the region bounded by $y = x^2$ and $y = 3x$.

Problem 13. Sketch the region of integration and change the order of integration.

(i) $\int_0^4 \int_{\sqrt{y}}^2 f(x, y)\, dxdy$

(ii) $\int_1^2 \int_0^{\ln x} f(x, y)\, dydx$

Problem 14. Set up but do not evaluate a double itegral that gives the volume of the solid under the surface $z = xy$ and above the triangle with vertices $(1, 1)$, $(1, 2)$ and $(2, 1)$

Problem 15. Evaluate $\int_0^2 \int_x^2 e^{-y^2}\, dydx$

Problem 16. Evaluate $\int_0^2 \int_{y^2}^4 \sqrt{x} \sin x\, dydx$

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Section 15.3

Recall: If $P(x, y)$ is a point in the xy-plane, we can represent the point P in polar form: Let r be the distance from O to P and let θ be the angle between the polar axis and the line OP. Then the point P is represented by the ordered pair (r, θ), and r, θ are called the **polar coordinates** of P.

Connecting polar coordinates with rectangular coordinates:

a.) $x = r\cos(\theta)$, $y = r\sin(\theta)$

b.) $\tan(\theta) = \frac{y}{x}$, thus $\theta = \arctan\left(\frac{y}{x}\right)$.

c.) $x^2 + y^2 = r^2$

Problem 1. Find the cartesian coordinates of the polar point $\left(2, \frac{2\pi}{3}\right)$.

Problem 2. Find the polar coordinates of the rectangular point $(\sqrt{3}, -1)$.

Problem 3. Find a cartesian equation for the curve described by $r = 2\sin\theta$.

Problem 4. Find a polar equation for $y = 1 + 3x$

Change to Polar Coordinates in a Double Integral: If f is continuous on a polar rectangle R given by $0 \leq a \leq r \leq b$, $\alpha \leq \theta \leq \beta$, where $0 \leq \beta - \alpha \leq 2\pi$, then

$$\iint_R f(x, y)\, dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta) r\,dr\,d\theta$$

Problem 5. Evaluate $\iint_R (x + 2)\, dA$, where R is the region bounded by the circle $x^2 + y^2 = 4$.

Problem 6. Evaluate $\iint_R 4y\, dA$, where R is the region in the second quadrant bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Problem 7. Evaluate $\iint_R 3x^2\, dA$, where R is the region in the first quadrant enclosed by the by the circle $x^2 + y^2 = 9$ and the lines $y = 0$ and $y = x$.

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Problem 8. Change $\int_0^3 \int_0^{\sqrt{9-x^2}} x^2 \, dy dx$ to a polar double integral. Do not evaluate.

Problem 9. Change $\int_0^4 \int_0^{\sqrt{4x-x^2}} \sqrt{x^2+y^2} \, dy dx$ to a polar double integral. Do not evaluate.

Problem 10. Set up but do not evaluate a double integral that gives the volume of the solid that lies above the xy-plane, below the sphere $x^2+y^2+z^2=81$ and inside the cylinder $x^2+y^2=4$

Problem 11. Find the volume of the solid bounded by the paraboloids $z = 20 - x^2 - y^2$ and $z = 4x^2 + 4y^2$.

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