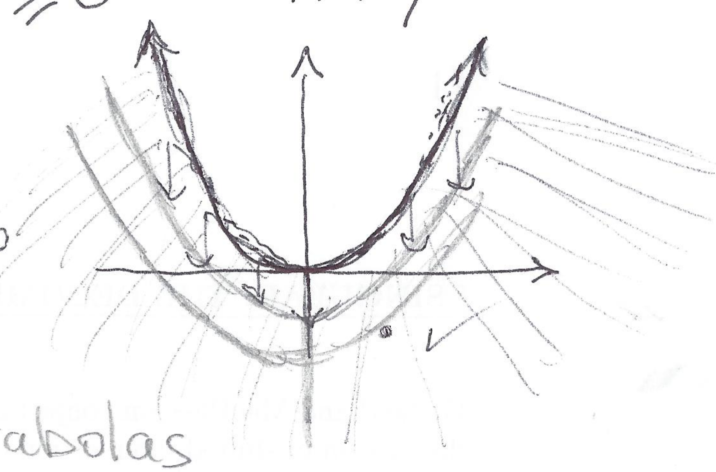
1. $f(x,y)=\sqrt{x^2-y}$

domain: $x^2-y \geq 0$ $\quad x^2 \geq y$

$y \leq x^2$



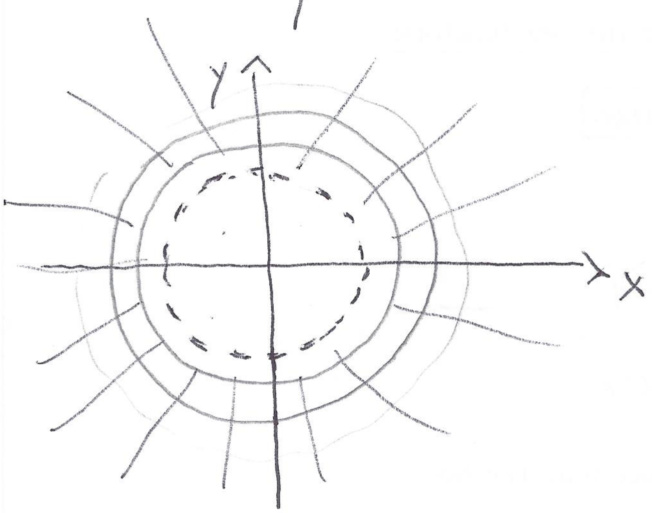
Level Curves: $\sqrt{x^2-y} = k \geq 0$

$x^2 - y = k^2$

$y = x^2 - k^2$ parabolas

2. $f(x,y) = \ln(y^2+x^2-1)$

$y^2+x^2-1 > 0$ $\quad x^2+y^2 > 1$



$e^{\boxed{\ln(y^2+x^2-1)}} = e^{k}$ → any real number

$y^2+x^2-1 = e^k$

$x^2+y^2 = e^k + 1 > 1$

circles radius > 1 c $(0,0)$

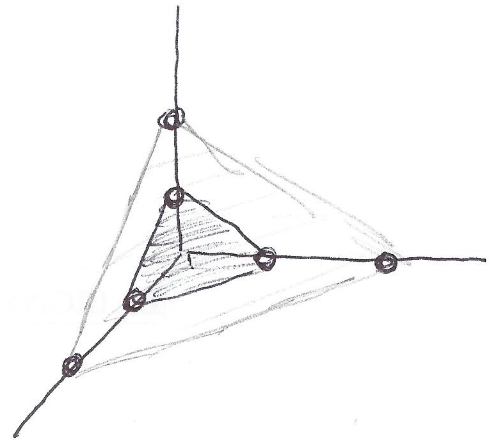
3. $f(x,y,z) = x + y + z = k$ plane

$k=1 \quad x+y+z=1$

$k=2 \quad x+y+z=2$

$\vec{n} = \langle 1, 1, 1 \rangle$

All planes with $\vec{n} = \langle 1, 1, 1 \rangle$



4. $f(x,y) = \sin(x^2+y^2)$

$f_x = 2x\cos(x^2+y^2)$

$f_{xx} = 2\cos(x^2+y^2) - (2x)^2\sin(x^2+y^2)$

$f_{xy} = 2x \cdot 2y(-\sin(x^2+y^2))$

$= -4xy\sin(x^2+y^2)$

same!! :)

$f_y = 2y\cos(x^2+y^2)$

$f_{yx} = -2y2x\sin(x^2+y^2)$

$f_{yy} = 2\cos(x^2+y^2) - 2y2y\sin(x^2+y^2)$

5. $z - z_0 = f_x(P_0)(x-x_0) + f_y(P_0)(y-y_0)$

$P_0(1,1)$

$z_0 = 2+1 = 3$

$f_x = 4x \quad 4$

$f_y = 2y \quad 2$

$z - 3 = 4(x-1) + 2(y-1)$

$$\boxed{4x + 2y - z - 3 = 0}$$

$-4-2+3$

6. $F(x,y,z) = 2xy + 3yz + 7xz + 9$

$4 - 6 - 7 = -9$

$P_0(1, 2, -1)$

$$F_x(P_0)(x - x_0) + F_y(P_0)(y - y_0) + F_z(P_0)(z - z_0) = 0$$

$F_x = 2y + 7z$ $\quad 4 - 7 = -3$

$F_y = 2x + 3z$ $\quad 2 - 3 = -1$

$F_z = 3y + 7x$ $\quad 6 + 7 = 13$

$$-3(x-1) - (y-2) + 13(z+1) = 0$$

$3 + 2 + 13$

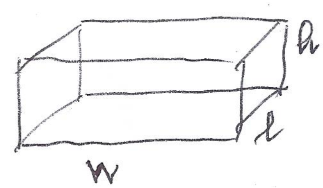
$$\boxed{-3x - y + 13z + 18 = 0}$$

7. $z = f(x,y) = x^3 y^2$ $\quad df = ?$

$df \approx \Delta f$

$$df = f_x\,dx + f_y\,dy$$

$$df = 3x^2y^2\,dx + 2x^3y\,dy$$



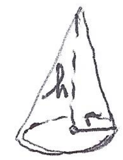
8. $A = 2(wl + lh + hw)$

$$dA = A_l\,dl + A_w\,dw + A_h\,dh$$

$$dA = 2(w+h)\,dl + 2(l+h)\,dw + 2(l+w)\,dh$$

9. $h = 3$ cm $dh = 0.1$ cm

$r = 2$ cm $dr = 0.2$ cm



$dV = ?$

$$V = \frac{1}{3}\pi r^2 h$$

$$dV = V_r\, dr + V_h\, dh$$

$$\frac{2r}{3}\pi h\, dr + \frac{1}{3}\pi r^2\, dh = \frac{4}{\cancel{3}}\pi \cancel{3} \cdot \frac{2}{10} + \frac{1}{3}\pi 4 \cdot \frac{1}{10} =$$

$$= \frac{8}{10}\pi + \frac{4}{30}\pi = \frac{24+4}{30}\pi = \frac{28}{30}\pi = \boxed{\frac{14}{15}\pi \text{ cm}^3}$$

10. Use linear approximation to estimate $(2.1^2 + 0.1^3)^3$

$f(x,y) = (x^2+y^3)^3$ $P_0(2,0)$

$$L(x,y) = z_0 + f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0)$$

$z_0 = (4+0)^3 = 4^3 = 64$

$$L(x,y) = 64 + 192(\overset{2.1}{x} - 2) + 0(y-0)$$

$$L(2.1, 0.1) = 64 + 192\,\frac{1}{10} =$$

$$= 64 + 19.2 = \boxed{83.2}$$

$\begin{array}{r}16\\ 4\\ \hline 64\end{array}$

$f_x = 3(x^2+y^3)^2(2x)$

$3(4^2) \cdot 4 = 192$

$f_y = 3(x^2+y^3)^2(3y^2)$ $\boxed{0}$

11. $f(x,y,z)=\sqrt{x^2+y^2+z^2}$ $\qquad P_0(3,2,6)$

$f_x = \dfrac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2+z^2}}$ $\qquad f_y = \dfrac{y}{\sqrt{\cdots}}$ $\quad \cdots \qquad \sqrt{9+4+36}=7$

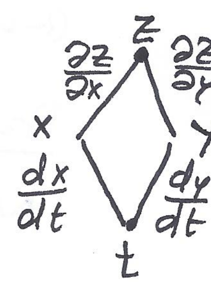
$L(x,y,z)=f(P_0)+f_x(P_0)(x-x_0)+f_y(P_0)(y-y_0)+f_z(P_0)(z-z_0)$

$$7+\frac{3}{7}(\overset{3.02}{x}-3)+\frac{2}{7}(\overset{1.97}{y}-2)+\frac{6}{7}(\overset{5.99}{z}-6)=$$

$$7+\frac{3}{7}\cdot\frac{2}{100}-\frac{2}{7}\frac{3}{100}-\frac{6}{7}\frac{1}{100}=7+\frac{\cancel{6}-\cancel{6}-6}{700}=7-\frac{6}{700}=$$

$$=\frac{4900-6}{700}=\frac{4894}{700}$$

12. $z=e^{x^2+y^2}$ $\qquad x=e^t \quad y=\cos t$

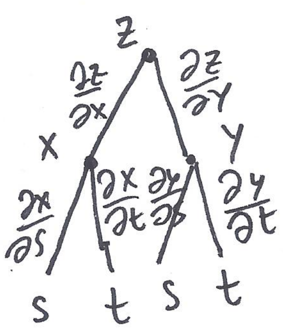


$$\frac{dz}{dt}=?=\frac{\partial z}{\partial x}\frac{dx}{dt}+\frac{\partial z}{\partial y}\frac{dy}{dt}=$$

$$=2xe^{x^2+y^2}(e^t)+2ye^{x^2+y^2}(-\sin t)=$$

$$=2e^{x^2+y^2}\left[xe^t-y\sin t\right]=2e^{e^{2t}+\cos^2 t}\left[e^{2t}-\cos t\sin t\right]$$

13. $z=xy$ $\qquad x=\cos(st^2)$ $\qquad y=\sin(e^t)$ $\qquad \dfrac{\partial z}{\partial t}=?\quad \dfrac{\partial z}{\partial s}=?$



$$\frac{\partial z}{\partial t}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

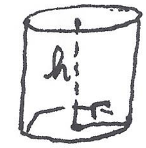
$$-y2st\sin(st^2)+xe^t\cos(e^t)$$

$$\frac{\partial z}{\partial s}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial s}+\underbrace{\frac{\partial z}{\partial y}\overset{=0}{\frac{\partial y}{\partial s}}}_{0}$$

$$-y\,t^2\sin(st^2)$$

14. $h = 1\,m \quad r = 2\,m$

$\frac{dh}{dt} = -2\,\frac{m}{s} \quad \frac{dr}{dt} = +4\,\frac{m}{s}$



$V = \pi r^2 h$

$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$

$(2\pi r h)(4) - \pi r^2 2 = 16\pi 1 - \pi 4 \cdot 2 = 16\pi - 8\pi = +8\pi\ \frac{m^3}{s}$

15. $D_{\vec{u}} f(P_0) = \vec{\nabla} f(P_0) \cdot \vec{u} \quad |\vec{u}| = 1$

$f(x,y) = \sqrt{xy} \quad P_0(4,1) \quad Q(6,2) \quad \vec{P_0Q} = Q - P_0 = \langle 2, 1 \rangle$

$|\vec{P_0Q}| = \sqrt{5}$

$\boxed{\vec{u} = \langle \frac{2}{\sqrt5}, \frac{1}{\sqrt5} \rangle}$

$\vec{\nabla} f = \langle \frac{y}{2\sqrt{xy}}, \frac{x}{2\sqrt{xy}} \rangle$

$\langle \frac{1}{4}, 1 \rangle \cdot \langle \frac{2}{\sqrt5}, \frac{1}{\sqrt5} \rangle = \frac{\cancel{2}\,1}{2\,\cancel{4}\sqrt5} + \frac{1}{\sqrt5} = \frac{3}{2\sqrt5}$

16. Direction of $\vec{\nabla} f(P_0) = \langle \frac{1}{4}, 1 \rangle \quad \sqrt{\frac{1}{16}+1} = \frac{\sqrt{17}}{4}$

Unit direction $= \langle (\sqrt{17})^{-1}, \frac{4}{\sqrt{17}} \rangle = \langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \rangle$

17. Max rate of change $= |\vec{\nabla} f(P_0)|$ $\quad f = e^{x+y} \quad P_0(-1,1)$

$\vec{\nabla} f = \langle e^{x+y}, e^{x+y} \rangle \quad \boxed{\langle 1, 1 \rangle}$

equals $|\langle 1,1 \rangle| = \sqrt{1+1} = \boxed{\sqrt{2}}$

18. $f(x,y) = 2x^3 - xy^2 + 5x^2 + y^2 + 5$

$\vec{\nabla} f = \vec{0}$ $\quad \langle 6x^2 - y^2 + 10x, -2xy + 2y \rangle$

$2y(1-x) = 0$ $\quad 6x^2 + 10x$ $\quad 6 - y^2 + 10$

$y = 0$ OR $x = 1$ $\quad 2x(3x+5) = 0$ $\quad 16 - y^2 = 0$

$x = 0 \quad x = -\frac{5}{3}$ $\quad y = \pm 4$

$(0,0)$ $\left(-\frac{5}{3}, 0\right)$ $(1,4)$ $(1,-4)$ CRIT. PTS.

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 12x+10 & -2y \\ -2y & -2x+2 \end{vmatrix}$$

$D(0,0) = \begin{vmatrix} \boxed{10} & 0 \\ 0 & 2 \end{vmatrix} = 20 > 0$ $\quad f_{xx} = 10 > 0$ $\quad (0,0)$ is a local min.

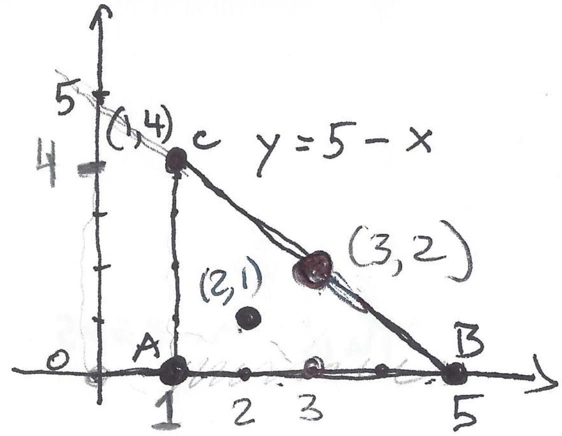
$+\frac{10}{3} + \frac{6}{3}$

$D\left(-\frac{5}{3}, 0\right) = \begin{vmatrix} -10 & 0 \\ 0 & +\frac{16}{3} \end{vmatrix} < 0$ $\quad \cancel{f_{xx} = -10 < 0}$ $\quad \left(-\frac{5}{3}, 0\right)$ is a ~~local max~~ sadde point

$D(1,4) = \begin{vmatrix} 22 & -8 \\ -8 & 0 \end{vmatrix} < 0$ $\quad (1,4)$ saddle pt.

$D(1,-4) = \begin{vmatrix} 22 & 8 \\ 8 & 0 \end{vmatrix} = < 0$ $\quad (1,-4)$ saddle pt.

19. $f(x,y) = 7 + xy - x - 2y$



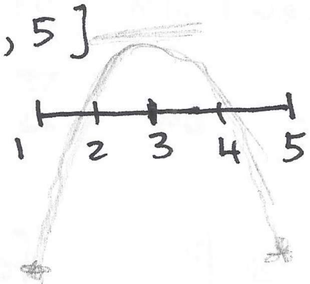
1. $\vec{\nabla} f = \vec{0}$: $\langle y-1,\ x-2 \rangle = \langle 0, 0 \rangle$
at $(2, 1)$

2. $\overline{AB}$ $y=0$ $f(x) = 7 - x$ on $[1, 5]$
$(1,0)$ $(5,0)$

$\overline{AC}$ $x = 1$ $f(y) =$

$= 7 + y - 1 - 2y =$ $7+4-1-8$
$= 6 - y$ on $[0, 4]$

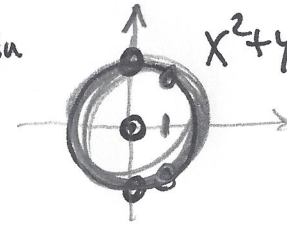
$\overline{BC}$: $y = 5 - x$
$f(x) = 7 + x(5-x) - x - 2(5-x)$
$7 + 5x - x^2 - x - 10 + 2x =$
$= -x^2 + 6x - 3$ on $[1, 5]$
$f'(x) = -2x + 6 = 0$ $x = 3$
$(3, 2)$



$f(2,1) = 7 + 2 - 2 - 2 =$
$= 5$
$* f(1,0) = 7 - 1 = 6 *$
$f(5,0) = 7 - 5 = 2$
$f(1,4) = 6 - 4 = 2$
$f(3,2) = 7 + 6 - 3 - 4 =$
$= 6$

Absolute min $= 2$ at $(5,0)$ & $(1,4)$
Absolute max $= 6$ at $(1,0)$ & $(3,2)$

20. $f = 2x^3 + y^4$ Absol. max/min



$x^2+y^2=1$

$y^2 = 1-x^2$

1: $\vec{\nabla} f = \langle 6x^2, 4y^3 \rangle = \vec{0}$ at $(0,0)$

$$\boxed{f(0,0) = 0}$$

2. $f = 2x^3 + (1-x^2)^2 = 2x^3 + 1 - 2x^2 + x^4 =$

$= x^4 + 2x^3 - 2x^2 + 1$

$f' = 4x^3 + 6x^2 - 4x = 2x(2x^2 + 3x - 2)$

$x = \dfrac{-3 \pm \sqrt{9+16}}{4} = \dfrac{-3 \pm 5}{4}$ → $\cancel{-2}$, $\dfrac{1}{2}$

$\underbrace{\left(\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right)}$ $\quad$ $\underbrace{\left(\tfrac{1}{2}, -\tfrac{\sqrt{3}}{2}\right)}$

$f = \dfrac{2}{8} + \dfrac{9}{16} = \dfrac{13}{16}$ $\quad$ $f = \dfrac{2}{8} + \dfrac{9}{16} = \dfrac{13}{16}$

Absolute Max $= 1$ at → $(0,1) \rightarrow f = 1$, $(0,-1) \rightarrow f = 1$

Absolute min $= 0$ at $(0,0)$.