Wir 6: Exam 2 Review

Sections 14.1, 14.3-14.8

**Problem 1.** Sketch the domain of $f(x,y) = \sqrt{x^2 - y}$ and describe the level curves.

**Problem 2.** Sketch the domain of $f(x,y) = \ln(y^2 + x^2 - 1)$ and describe the level curves.

**Problem 3.** What are the level surfaces to the equation $f(x,y,x) = x + y + z$?

**Problem 4.** $f(x,y) = \sin(x^2 + y^2)$, find all first and second partial derivatives.

**Problem 5.** Find an equation for the tangent plane to the surface $z = 2x^2 + y^2$ at the point $(1,1)$

**Problem 6.** Find the tangent plane to the surface $2xy + 3yz + 7xz = -9$ at the point $(1,2,-1)$.

**Problem 7.** If $z = x^3y^2$, find the differential, $dz$, and explain what it measures.

**Problem 8.** Consider a rectangular box with length $l$, width $w$ and height $h$. If $A$ is the surface area of the box, find the differential, $dA$.

**Problem 9.** The height of a cone was measured to be 3 cm with a maximum error of 0.1 cm and the radius of the cone is measured to be 2 cm with a maximum error of 0.2 cm. Use differentials to estimate the maximum error in the calculated volume of the cone.

**Problem 10.** Use a linear approximation (tangent plane) to estimate $\left((2.1)^2 + (0.1)^3\right)^3$

**Problem 11.** Use differentials to approximate $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$

*With thanks to Amy Austin for generously sharing all of her WIR problems from last semester.*

**Problem 12.** If $z = e^{x^2+y^2}$, $x = e^t$. $y = \cos t$, find $\frac{dz}{dt}$

**Problem 13.** For $z = xy$, $x = \cos(st^2)$, $y = \sin e^t$, find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$.

**Problem 14.** The radius and height of a circular cylinder change with time. When the height is 1 meter and the radius is 2 meters, the radius is increasing at a rate of 4 meters per second and the height is decreasing at a rate of 2 meters per second. At that same instant, find the rate at which the volume is changing

**Problem 15.** Let $f(x,y) = \sqrt{xy}$. Find the directional derivative of $f$ at the point $P(4,1)$ in the direction from $P$ to $Q(6,2)$

**Problem 16.** Let $f(x,y) = \sqrt{xy}$. What is the direction of the largest rate of change at the point $P(4,1)$?

**Problem 17.** Let $f(x,y) = e^{x+y}$. What is the maximum rate of change at the point $P(-1,1)$?

**Problem 18.** For the $f(x,y) = 2x^3 - xy^2 + 5x^2 + y^2 + 5$, find all local minima, maxima, and saddle points.

**Problem 19.** Find the absolute maximum and minimum values of $f(x,y) = 7 + xy - x - 2y$ over the closed triangular region with verticies $(1,0)$, $(5,0)$, $(1,4)$.

**Problem 20.** Find the absolute maximum and minimum values of $f(x,y) = 2x^3 + y^4$ over the region $D = \{(x,y) : x^2 + y^2 \leq 1\}$

**Problem 21.** Use the method of Lagrange to find the maximum and minimum values of $f(x,y) = 6x + 6y$ subject to the constraint $x^2 + y^2 = 18$.

**Problem 22.** Use the method of Lagrange to find the maximum and minimum values of $f(x,y) = y^2 - x^2$ subject to the constraint $\frac{1}{4}x^2 + y^2 = 25$

**Problem 23.** Find the volume of the largest rectangular box with faces parallel to the coordinate planes than can be inscribed in the ellipsoid $16x^2 + 4y^2 + 9z^2 = 144$.

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