WIR 4

$f(x,y)$ dom inside of $\mathbb{R}^2$

Section 14.1

Problem 1. Find and sketch the domain of the following functions.

a.) $f(x,y) = \sqrt{4x - 2y}$

b.) $f(x,y) = \ln(y - 3x)$

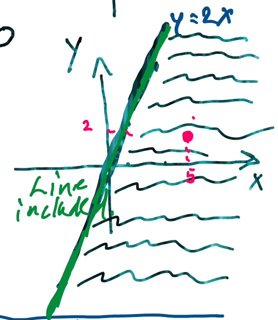
c.) $f(x,y) = \sqrt[4]{xy}$

d.) $f(x,y) = \dfrac{\sqrt{8 - x^2 - y^2}}{x + 2y}$

e.) $f(x,y) = \dfrac{1}{\sqrt{x + 2y}} + \sqrt{4 - x^2 - y^2}$

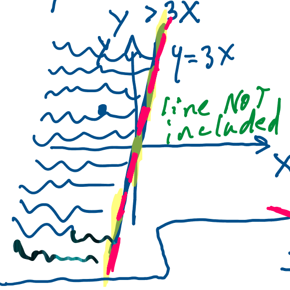
a) $4x - 2y \geq 0$

$2x \geq y$



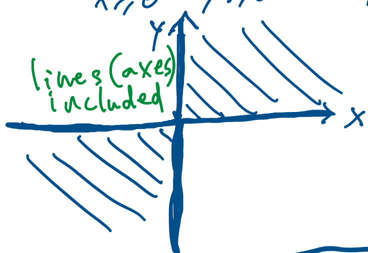
b) $y - 3x > 0$

$y > 3x$



c) $xy \geq 0$

$x \geq 0 \quad y \geq 0$ OR $x \leq 0 \quad y \leq 0$

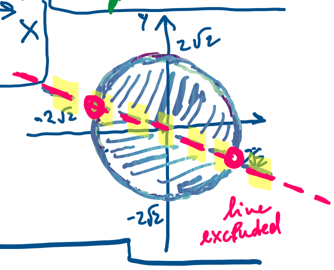


d) $8 - x^2 - y^2 \geq 0$

$8 \geq x^2 + y^2$

$x + 2y \neq 0$

$x \neq -2y$

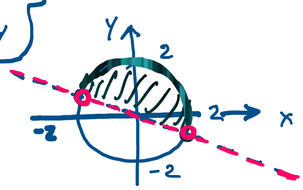


e) $4 - x^2 - y^2 \geq 0$

$4 \geq x^2 + y^2$

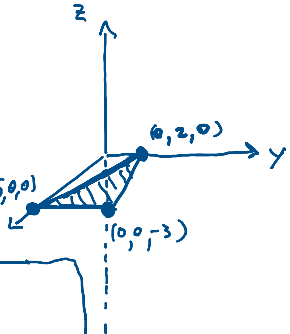
$x + 2y > 0$

$y > -\frac{x}{2}$



Problem 2. Sketch the graph of the following surfaces:

a.) $2z = x + 3y - 6$ ⓐ $x + 3y - 2z = 6$

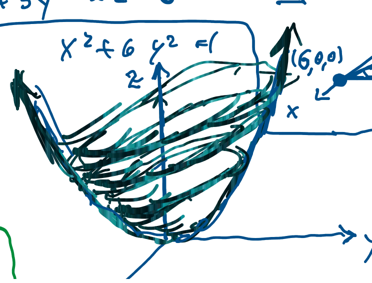


b.) $z = x^2 + 6y^2$

$\boxed{\text{c.) } z = y^2}$

d.) $z = 7 - x^2 - 2y^2$

$z = 1$, $x^2 + 6y^2 = 1$

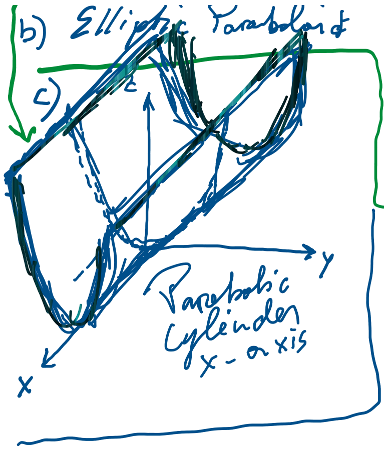


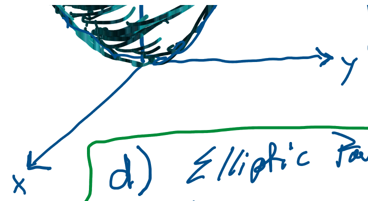
b) Elliptic Paraboloid

c)

b) Elliptic Paraboloid

c)

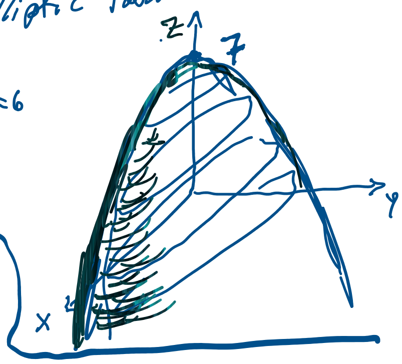




d) Elliptic Paraboloid.

$z=1$

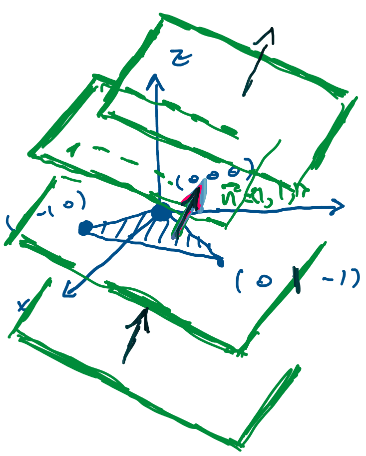
$x^2+2y^2=7-1=6$



_ Problem 3 at the bottom of this page _

Problem 4. Describe the level surfaces of $f(x,y,z)=x+y+z$.

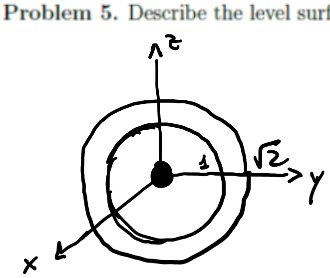
Set $f(x,y,z)=k$



$x+y+z = 2,\ 1,\ 0,\ -1,\ -2$

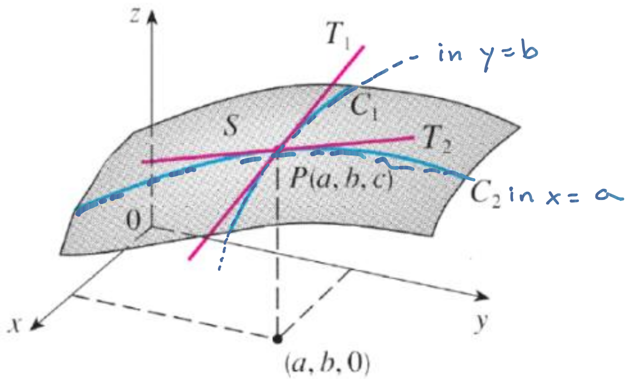
$\vec{n}=\langle 1,1,1\rangle$

Problem 5. Describe the level surfaces of $f(x,y,z)=x^2+y^2+z^2 = 1,\ 0,\ 2$



spheres centered at the origin.

Section 14.3



Problem 6. Find $f_x(-1, 2)$ and $f_y(-1, 2)$ for $f(x, y) = x^3 - y^4 - 6x^2y^3$

$$\frac{\partial f}{\partial x} = f_x = 3x^2 - 0 - 12xy^3$$

$$f_x(-1,2) = 3 + 12 \cdot 8 = 3 + 96 = 99$$

$$\frac{\partial f}{\partial y} = f_y = 0 - 4y^3 - 18x^2y^2$$

$$f_y(-1,2) = -32 - 18 \cdot 1 \cdot 4 = -32 - 72 =$$

$$= -104$$

Problem 7. Find $f_x(x,y)$ and $f_y(x,y)$ for $f(x,y) = x^2 e^{\cos(2x^4y^2)}$

$$f_x = 2x\, e^{\cos(2x^4y^2)} + x^2 e^{\cos(2x^4y^2)}\left[-\sin(2x^4y^2)\right](8x^3y^2) =$$

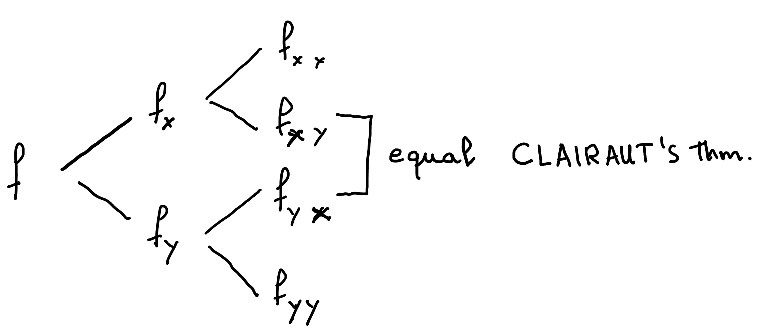
$$= e^{\cos(2x^4y^2)}\left[2x - 8x^5y^2 \sin(2x^4y^2)\right]$$

$$f_y = (x^2)\, e^{\cos(2x^4y^2)}\left[-\sin(2x^4y^2)\right](4x^4y)$$

Problem 8. If $f(x,y) = ye^{-x} + 2x$, find $\left.\frac{\partial f}{\partial x}\right|_{(1,0)}$ and $\left.\frac{\partial f}{\partial y}\right|_{(1,0)}$

$$\frac{\partial f}{\partial x} = -ye^{-x} + 2 \quad \text{at } (1,0) = 2$$

$$\frac{\partial f}{\partial y} = e^{-x} + 0 \quad \text{at } (1,0) = e^{-1} \text{ or } \frac{1}{e}$$



$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$$

In problem 9, "higher order" means "up to order two".

Problem 9. Find all higher order partial derivatives for $f(x,y) = \ln(2x+3y)$

$$f_x = \frac{2}{2x+3y} \qquad f_{xx} = \{2\}\,\frac{-1}{(2x+3y)^2}\cdot 2 = \frac{-4}{(2x+3y)^2}$$

$f_{xx} = (2) \frac{-1}{(2x+3y)^2} \cdot 2 = \frac{\;}{(2x+3y)^2}$

$f_x = \frac{2}{2x+3y}$

$f_{xy} = \frac{-6}{(2x+3y)^2}$

f

SAME

$f_{yx} = \frac{-6}{(2x+3y)^2}$

$f_y = \frac{3}{2x+3y}$

$f_{yy} = \frac{-9}{(2x+3y)^2}$

Section 14.4

Problem 10. Find the differential of $z = x^2 + 2y^2 + 4xy$ at the point $(1, 2)$.

$dz = z_x \, dx + z_y \, dy$ $\quad$ z_x and z_y must be evaluated at $(1,2)$.

$z_x = \frac{\partial z}{\partial x} = 2x + 0 + 4y$ at $(1,2)$ it equals $2 + 8 = 10$

$z_y = \frac{\partial z}{\partial y} = 0 + 4y + 4x$ $\quad$ " $\quad$ " $\quad$ " $\quad$ " $\quad$ $8 + 4 = 12$

$dz = 10\,dx + 12\,dy$ at point $(1, 2)$.

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Problem 11. Find the differential of $f(x,y,z) = x^2y^3z^4$.

$df = f_x\,dx + f_y\,dy + f_z\,dz$

$f_x = 2xy^3z^4$ $\quad f_y = 3x^2y^2z^4$ $\quad f_z = 4x^2y^3z^3$

$df = 2xy^3z^4\,dx + 3x^2y^2z^4\,dy + 4x^2y^3z^3\,dz$

Problem 12. Find an equation of the tangent plane to the surface $z = x^3 - 3y^2$ at point $(-1, 1)$.

$z - z_0 = f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0)$

$z_0 = z(-1, 1) = -1 - 3 = -4$

$f_x = z_x = 3x^2$ at $(-1, 1)$ it is 3

$f_y = -6y$ at $(-1, 1)$ it is -6

$z + 4 = 3(x+1) - 6(y-1)$ OR

$3 + 6 - 4 = 5$

$$\boxed{3x - 6y - z + 5 = 0}$$

Problem 13. Find an equation of the tangent plane to the surface $z = e^{x-y}$ at point $(2, 2, 1)$. What is the equation of the normal line to this tangent plane at point $(2, 2, 1)$?

Note: $z_0 = z(2,2) = e^{2-2} = e^0 = 1$ ✓

$z_x = e^{x-y}(1)$

$z_x(2,2) = e^0 = 1$

$z_y = -e^{x-y}$; $z_y(2,2) = -e^0 = -1$

$z - 1 = 1(x-2) - 1(y-2)$ OR

$-\cancel{2} + \cancel{2} + 1$

$$\boxed{x - y - z + 1 = 0}$$

Problem 14. Using the tangent plane to the graph of $f(x, y) = \sqrt{24 - x^2 - y^2}$ at point $(2, 2)$, approximate $f(2.09, 1.93)$.

$\sqrt{24-4-4} = 4 = f(P_0)$

$L(x,y) = f(P_0) + f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0)$

$f_x = \dfrac{-\cancel{2}x}{\cancel{2}\sqrt{24-x^2-y^2}} = \dfrac{-x}{\sqrt{24-x^2-y^2}}$ at $(2,2)$ it is $\frac{-2}{4} = -\frac{1}{2}$

$f_y = \dfrac{-y}{\sqrt{24-x^2-y^2}}$ at $(2,2)$ it is $\frac{-2}{4} = -\frac{1}{2}$

$798 \; \frac{399}{2}$

$L(x,y) = 4 - \frac{1}{2}(x-2) - \frac{1}{2}(y-2)$

$L(2.09, 1.93) = 4 - \frac{1}{2}(0.09) - \frac{1}{2}(-0.07) =$

$\dots \frac{7}{\dots} = \frac{800-9+7}{200} = \frac{798}{200} =$

$L(2.09, 1.45) = \ldots$

$$= \frac{8}{2}\,\frac{100}{100} - \frac{1}{2}\,\frac{9}{100} + \frac{1}{2}\,\frac{7}{100} = \frac{800-9+7}{200} = \frac{798}{200} =$$

$$= \frac{399}{100} = \boxed{3.99}$$

$(8-2+4)^3 = 10^3 = 1000$ $\quad P_0(2,1,1)$

$f(P_0) = 1000$

Problem 15. Use differentials to approximate $((1.97)^3 - 2(0.9)^4 + 4(1.01)^5)^3$.

We need a $f(x,y,z)\ldots$ and a "easy point" $P_0\ldots$

$f(x,y,z) = (x^3 - 2y^4 + 4z^5)^3$

$f_x = 3(x^3 - 2y^4 + 4z^5)^2(3x^2)$ at $P_0 \to 3\cdot 100\cdot 3\cdot 4 = 3600$

$f_y = 3(x^3 - 2y^4 + 4z^5)^2(-8y^3) = 3\cdot 100\cdot(-8) = -2400$

$f_z = 3(x^3 - 2y^4 + 4z^5)^2(20z^4) = 3\cdot 100(20) = 6000$

$L(1.97, 0.9, 1.01) = 1000 + 3600(1.97-2) - 2400(0.9-1) + 6000(1.01-1) =$

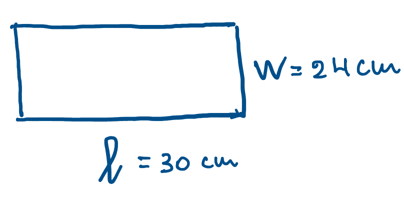
$= 1000 + 36\cancel{00}\left(-\frac{3}{\cancel{100}}\right) - 240\cancel{0}\left(-\frac{1}{\cancel{10}}\right) + 60\cancel{00}\left(\frac{1}{\cancel{100}}\right) =$

$= 1000 - 108 + 240 + 60 = 1300 - 108 = \boxed{1292}$

Problem 16. The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error in measurement of 0.1 cm in both. Use differentials to approximate the maximum error in the calculated area of the rectangle.

$d\ell = dw = 0.1$ cm

$dA = ?$ $\quad A = \ell w$



$dA = A_\ell\, d\ell + A_w\, dw =$

$= w\, d\ell + \ell\, dw = 24\,\frac{1}{10} + 30\,\frac{1}{10} = 2.4 + 3 = \underline{5.4\ \text{cm}^2}$

$\ldots 720\ \text{cm}^2)$

$= w\, dl + l\, dw = \ldots 10 \qquad \ldots 10$

$\left(\text{compare to } A = 24 \cdot 30 = 720 \text{ cm}^2\right)$

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Pb 3: Find level curves of

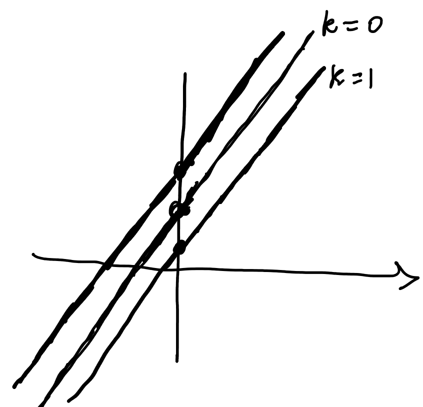
a) $f(x,y) = 2 + 4x - y = 0$

$1 \cdot\; y = 4x + 1$

$-1 \cdot\; y = 4x + 3$

$4x - y = -2$

$4x + 2 = y$

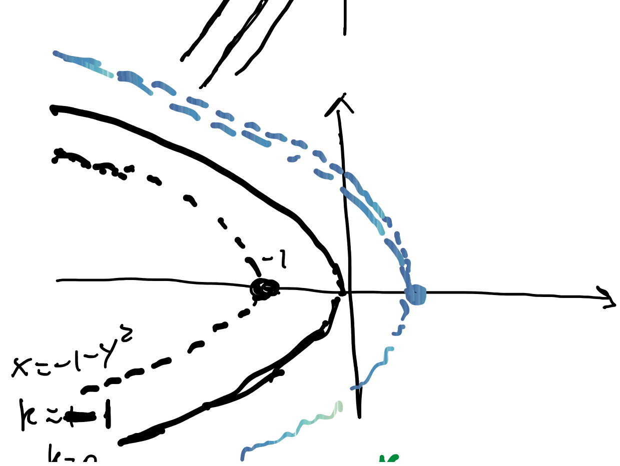


b) $f(x,y) = x + y^2 = 0$

$1 \cdot\; \boxed{x = 1 - y^2}$

$-1 \cdot\; x = -1 - y^2$

$x = -y^2$



$9 - x^2 - y^2 = 4$

$x^2 + y^2 = 5$

$x^2 + y^2 = 8$

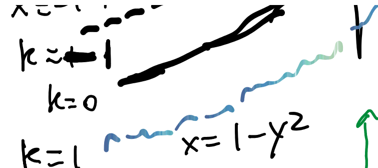
$9 - x^2 - y^2 = 1$

1.

$9 - x^2 - y^2 = \ldots$

$x^2 + y^2 = 5$

$9 - x^2 - y^2 = 1$

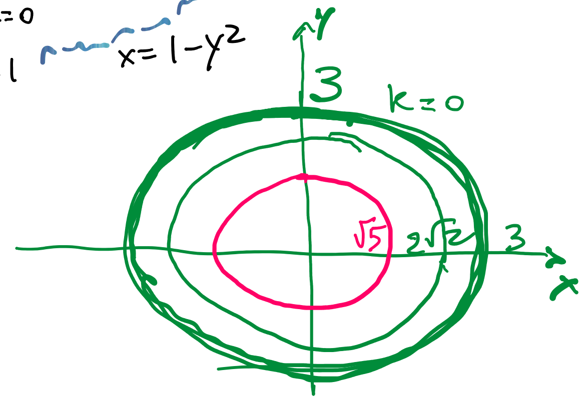


c) $f(x,y) = \sqrt{9 - x^2 - y^2} = \begin{cases} 0 \\ 1 \\ 2 \\ \cancel{4} \end{cases}$

$x^2 + y^2 = 9$

$\cancel{9 - x^2 - y^2 = 16}$

$\cancel{x^2 + y^2 = -7}$



d) $f(x,y) = 8\sqrt{x^2 - y^2} = \begin{cases} 0 \\ 1 \\ 2 \end{cases}$

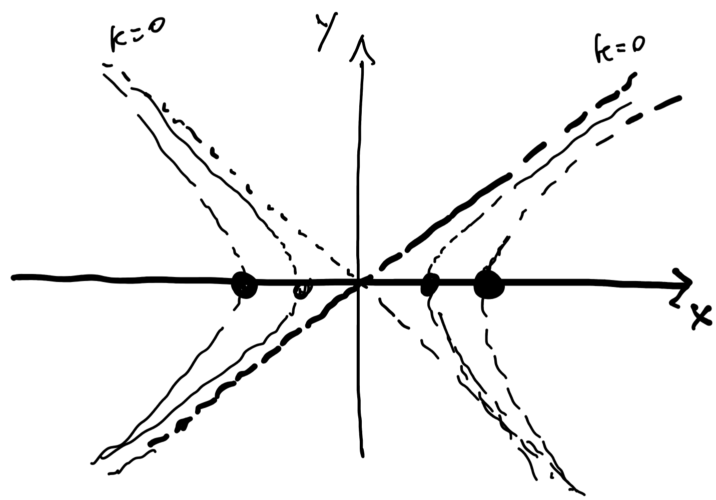
$x^2 = y^2 \Rightarrow x = \pm y$

$64(x^2 - y^2) = 1$

$x^2 - y^2 = \frac{1}{64}$

$x^2 = y^2 + \frac{1}{64}$

$x = \pm\sqrt{y^2 + \frac{1}{64}}$



$64(x^2 - y^2) = 4$

$x^2 - y^2 = \frac{4}{64}$

$x^2 = \frac{4}{64} + y^2$

$x = \pm\sqrt{\frac{4}{64} + y^2}$

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