Wir 2

Tuesday, February 2, 2021 3:43 PM

Wir 2: 12.4, 12.5, 12.6

Section 12.4

1. Find the cross product of $\overbrace{\langle 1,1,3\rangle}^{\vec{a}}$ and $\overbrace{\langle -2,-1,5\rangle}^{\vec{b}}$ and find the area of the parallelogram determined by the two vectors.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ -2 & -1 & 5 \end{vmatrix} = \underbrace{\begin{vmatrix} 1 & 3 \\ -1 & 5 \end{vmatrix}}_{5-(-3)}\hat{i} - \underbrace{\begin{vmatrix} 1 & 3 \\ -2 & 5 \end{vmatrix}}_{-(5+6)}\hat{j} + \underbrace{\begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix}}_{-1+2}\hat{k} =$$

$$= 8\hat{i} - 11\hat{j} + 1\hat{k}$$

$8-11+3$ ✓

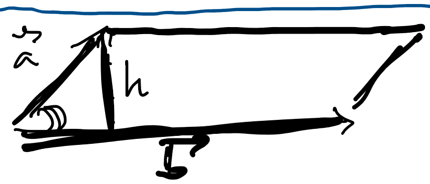
$-16+11+5$ ✓

$$\text{Area} = |\vec{a}\times\vec{b}| = \sqrt{64+121+1} = \sqrt{186}$$

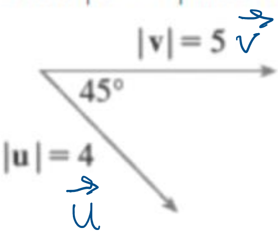
121
65

186 | 2
93 | 3
31 | 31
1 | 1

$$|\vec{a}\times\vec{b}| = |a|\,|b|\sin\theta$$



2. Find $|\mathbf{u}\times\mathbf{v}|$ and determine if $\mathbf{u}\times\mathbf{v}$ points in or out of the page.



$$|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\theta = 4\cdot 5\sin 45^\circ =$$

$$= 20\,\frac{\sqrt{2}}{2} = 10\sqrt{2}$$

3. Find two unit vectors that are orthogonal to the plane that passes through the points $P(1,0,1)$, $Q(2,3,4)$ and $R(2,1,1)$.

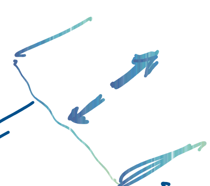
$$\vec{PQ} = Q-P = \langle 1, 3, 3 \rangle$$

$$\vec{PR} = R-P = \langle 1, 1, 0 \rangle$$

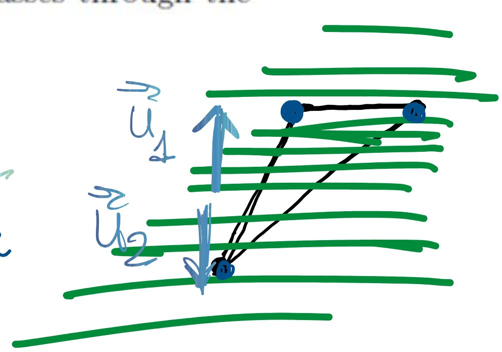
with $\hat{i}\ \hat{j}\ \hat{k}$ above the columns.

$-3+9-6$ ✓

$-3+3$ ✓



$$\langle -3, -(0-3), -2\rangle = \langle -3, 3, -2\rangle = \vec{PQ}\times\vec{PR}$$



$\langle -3, -(0-3), -2 \rangle = \langle -3, 3, $

$$\boxed{\begin{aligned} \vec{u}_1 &= \left\langle \frac{-3}{\sqrt{22}}, \frac{3}{\sqrt{22}}, \frac{-2}{\sqrt{22}} \right\rangle \\ \vec{u}_2 &= \left\langle \frac{3}{\sqrt{22}}, \frac{-3}{\sqrt{22}}, \frac{2}{\sqrt{22}} \right\rangle \end{aligned}}$$

$|\langle -3, 3, -2 \rangle| =$

$= \sqrt{9+9+4} = \sqrt{22}$

4. Determine whether each expression is meaningful or meaningless (circle one). If so, state whether the expression is a vector or a scalar.

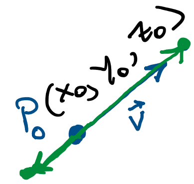
a.) $\mathbf{a} \cdot \mathbf{b}$	meaningful (vector or scalar)	meaningless
b.) $\mathbf{a} \times \mathbf{b}$	meaningful (vector or scalar)	meaningless
c.) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$	meaningful (vector or scalar)	meaningless
d.) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$	meaningful (vector or scalar)	meaningless
e.) $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$	meaningful (vector or scalar)	meaningless
f.) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$	meaningful (vector or scalar)	meaningless
g.) $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$	meaningful (vector or scalar)	meaningless
h.) $\lvert\mathbf{a}\rvert(\mathbf{b} \times \mathbf{c})$	meaningful (vector or scalar)	meaningless

Section 12.5

1. Find vector, parametric, and symmetric equations for the line through the point $\overbrace{(1, 0, -3)}^{P_0}$ and parallel to the vector $2\,\boldsymbol{i} - 4\,\boldsymbol{j} + 5\,\boldsymbol{k}$. $= \vec{v}$

$$\boxed{\vec{r}(t) = \vec{r}_0 + t\,\vec{v}}$$

$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$



$\langle x, y, z \rangle = \langle 1, 0, -3 \rangle + t\langle 2, -4, 5 \rangle \Rightarrow$ vector equ.

$$\begin{cases} x = 1 + 2t \\ y = 0 - 4t \\ z = -3 + 5t \end{cases}$$ Parametric equs.

$$t = \boxed{\frac{x-1}{2} = -\frac{y}{4} = \frac{z+3}{5}}$$ symmetric equs.

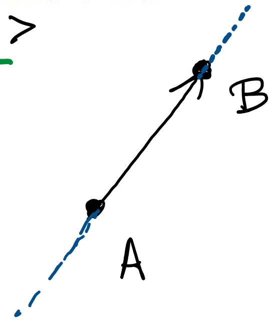
2. Find parametric and symmetric equations of the line through the points $\overbrace{(1, 2, 0)}^{A}$ and $\overbrace{(-5, 4, 2)}^{B}$.

$P_0 = A$ or B

$\vec{v} = ?\quad B - A = \vec{AB} = \langle -6, 2, 2 \rangle$

2. Find parametric and symmetric equations of the line through the points (1, 2, 0) and (-5, 4, 2).

$P_0 = A$ or B $\vec{v} = ?$ $B - A = \vec{AB} = \langle -6, 2, 2 \rangle$



$\langle x, y, z \rangle = \langle 1, 2, 0 \rangle + t \langle -6, 2, 2 \rangle$ V.E.

$\begin{cases} x = 1 - 6t \\ y = 2 + 2t \\ z = 0 + 2t \end{cases}$ P.E. OR $\begin{cases} x = -5 - 3s \\ y = 4 + s \\ z = 2 + s \end{cases}$

$t = \dfrac{x-1}{-6} = \boxed{\dfrac{1-x}{6} = \dfrac{y-2}{2} = \dfrac{z}{2}}$ S.E.

3. Find parametric and symmetric equations of the line passing through the point $\underbrace{(-3, 5, 4)}_{P_0}$ and parallel to the line $x = 1 + 3t$, $y = -1 - 2t$, $z = 3 + t$. $\vec{V} = \langle 3, -2, 1 \rangle$

$\begin{cases} x = -3 + 3t \\ y = 5 - 2t \\ z = 4 + t \end{cases}$ P.E.

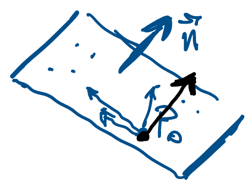
$t = \boxed{\dfrac{x+3}{3} = \dfrac{y-5}{-2} = z - 4}$ S.E.

$\dfrac{5-y}{2}$

$\begin{cases} \dfrac{x+3}{3} = \dfrac{5-y}{2} \text{ plane} \\ \dfrac{5-y}{2} = z - 4 \text{ plane} \end{cases}$

4. Find an equation of the plane through the point $\underbrace{(-4, 3, 1)}_{P_0}$ that is perpendicular to the vector $\vec{a} = -4\mathbf{i} + 7\mathbf{j} - 2\mathbf{k} = \vec{n}$

$\boxed{\vec{n} \cdot \vec{P_0 P} = 0}$ $P(x, y, z)$



$\langle -4, 7, -2 \rangle \cdot \langle x+4, y-3, z-1 \rangle = 0$

$-4(x+4) + 7(y-3) - 2(z-1) = 0$ $+16 + 21 - 2$

$\boxed{-4x + 7y - 2z - 35 = 0}$

21
14

A B C

5. Find an equation of the plane passing through the points $\underbrace{(1, 2, -3)}_{A}$, $\underbrace{(2, 3, 1)}_{B}$, and $\underbrace{(0, -2, -1)}_{C}$.

$P_0 = A$ or B or C

$\vec{n} = \vec{CA} \times \vec{CB} = (A-C)\times(B-C) =$

$= \langle 1, 4, -2 \rangle \times \langle 2, 5, 2 \rangle =$

$\begin{vmatrix} i & j & k \\ 1 & 4 & -2 \\ 2 & 5 & 2 \end{vmatrix} = \langle 18, -6, -3 \rangle = \vec{n}$

$P_0 = C$

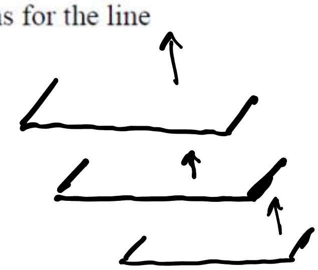
$\cancel{18}\,6(x-0) - \cancel{6}\,2(y+2) - \cancel{3}\,1(z+1) = 0$

$6x - 2y - z = 4 + 1 = 5$

$6x - 2y - z - 5 = 0$

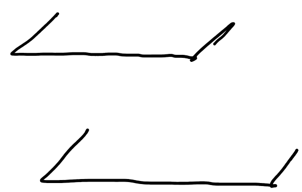
6. Determine whether the planes $3x + y - 4z = 3$ and $-9x - 3y + 12z = 4$ are orthogonal, parallel, or neither. Find the angle of intersection and the set of parametric equations for the line of intersection of the planes.

$\vec{n}_1 = \langle 3, 1, -4 \rangle$ $\quad \vec{n}_2 = \langle -9, -3, 12 \rangle$



$\underbrace{\vec{n}_1 \parallel \vec{n}_2}_{\text{multiples.}} \rightarrow$ planes are parallel.

NO INTERSECTION



$\boxed{\begin{array}{l} 2x + y - 7z \\ 2x + y - 7z \end{array}}\begin{array}{l} = 5 \\ = 28 \end{array}$

7. Determine whether the planes $x - 3y + 6z = 4$ and $5x + y - z = 4$ are orthogonal, parallel, or neither. Find the angle of intersection and the set of parametric equations for the line of intersection of the planes.

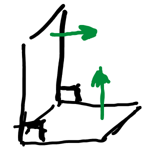
7. Determine whether the planes $x - 3y + 6z = 4$ and $5x + y - z = 4$ are orthogonal, parallel, or neither. Find the angle of intersection and the set of parametric equations for the line of intersection of the planes.

$\vec{n}_1 = \langle 1, -3, 6 \rangle \qquad \vec{n}_2 = \langle 5, 1, -1 \rangle$

NOT //.

$\vec{n}_1 \bullet \vec{n}_2 = 5 - 3 - 6 \neq 0 \quad = -4$

so not $\perp$



$$\theta = \cos^{-1}\left(\frac{|\vec{n}_1 \bullet \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}\right) = \cos^{-1}\left(\frac{4}{\sqrt{46}\sqrt{27}}\right)$$

degrees ✓ rad. ✓

decimal places ✓

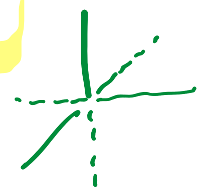
$\sqrt{1+9+36} = \sqrt{46}$

$\sqrt{25+1+1} = \sqrt{27}$

$$\begin{cases} x - 3y + 6z = 4 \\ 5x + y - z = 4 \end{cases} \quad (3)$$

$P_0(1, -1, 0)$

$15x + 3y - 3z = 12$



$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 1 & -1 \\ 1 & -3 & 6 \end{vmatrix} =$$

$16x + 3z = 16$

$z = 0$ then $x = 1$

$5 + y = 4$

$y = -1$

$= \langle 3, -(31), -16 \rangle = \vec{v}$

$$\begin{cases} x = 1 + 3t \\ y = -1 - 31t \\ z = 0 - 16t \end{cases}$$

=0

8. Find the point where the line $x = 1 + t$, $y = 2t$, and $z = -3t$ intersects the plane with equation $-4x + 2y - 4z = -2$.

$-4(1+t) + 2(2t) - 4(-3t) = -2$

$-4 - \cancel{4t} + \cancel{4t} + 12t = -2 \rightarrow 12t = 2$

$t = \frac{1}{6}$

$x = 1 + \frac{1}{6} = \frac{7}{6} \qquad y = 2\frac{1}{6} = \frac{1}{3} \qquad z = -3\frac{1}{6} = -\frac{1}{2}$

$\left(\frac{7}{6}, \frac{1}{3}, -\frac{1}{2}\right)$

9. Find the distance between point (1, 2, 3) and the plane with equation $2x - y + z \cancel{=} - 4 = 0$

$$\frac{|2x - y + z - 4|}{\sqrt{2^2+(-1)^2+1^2}} = \frac{|\cancel{2} - \cancel{2} + 3 - 4|}{\sqrt{4+1+1}} = \boxed{\frac{1}{\sqrt{6}}}$$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Ellipsoid

$z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ Ellipt. Paraboloid

$z^2=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ cone

$z=\frac{x^2}{a^2}-\frac{y^2}{b^2}$ Hyperb. Parabol.

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ 1-sheeted Hyp.

$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ 2-sheeted Hyp

Section 12.6

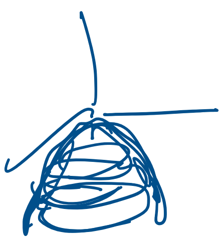
1. Identify and sketch the following quadric surfaces:

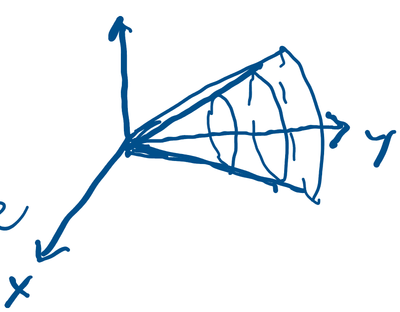
a) $z = (x + 4)^2 + (y - 2)^2 + 5.$ Elliptic Paraboloid

$Z = X^2 + Y^2 + 5$

V (-4, 2, 0)

b) $z = -(x^2 + y^2)$ Elliptic Paraboloid





c) $y^2 = x^2 + z^2$ cone

d) $x^2 + y^2 + z - 4x - 6y + 13 = 0.$

$x^2 - 4x + 4 + y^2 - 6y + 9 + z + 13 = +4 + 9$

$(x-2)^2 + (y-3)^2 + z = 0$ Elliptic Paraboloid

$(x-2)^2+(y-3)^2+z=0$

Elliptic paraboloid

opens down

$z=-(x^2+y^2)$

$V(2,3,0)$