Week 8 Week in Review

courtesy: David J. Manuel

(covering 11.2, 11.3, and Exam II Review)

1 Section 11.2

1. Determine if the following series converge or diverge. If they converge, find their sum.

   (a) $\displaystyle\sum_{n=1}^{\infty}\left(\cos\left(\frac{1}{n+2}\right)-\cos\left(\frac{1}{n}\right)\right)$

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{3n}{5n+4}$

   (c) $\displaystyle\sum_{n=0}^{\infty}\frac{(-2)^n+3^{2n}}{10^n}$

2. The series $\displaystyle\sum_{n=1}^{\infty}a_n$ has partial sums given by $s_N=\dfrac{N}{3N+1}$.

   (a) Find a_{2023} (you do NOT have to simplify your answer)

   (b) Determine if the series converges or diverges. If it converges, find the sum.

2 Exam II Review

1. Evaluate the following integrals:

   (a) $\displaystyle\int(\tan^3 x)(\sec^5 x)\,dx$

   (b) $\displaystyle\int\frac{dx}{x^4\sqrt{x^2-4}}$

   (c) $\displaystyle\int\frac{7x^2+3x+11}{(x+1)(x^2+4)}\,dx$

   (d) $\displaystyle\int_{-\infty}^{0}ze^{3z}\,dz$

   (e) $\displaystyle\int_0^4\frac{dx}{\sqrt{x}}$

2. Determine whether $\displaystyle\int_1^\infty \frac{dx}{x - e^{-5x}}$ converges or diverges (you do NOT have to compute the integral if it converges).

3. Determine whether the following sequences converge or diverge. If a sequence converges, find the limit; if a sequence diverges, explain why.

   (a) $a_n = \ln(100n^2 + 5) - 2\ln(5n + 1)$

   (b) $a_n = \dfrac{(-1)^n 3n}{n + 1}$

   (c) $a_n = \dfrac{(-1)^n 3}{n + 1}$

   (d) $a_n = (\sqrt{n^2 + 5n + 1} - n)$

4. It can be shown that the sequence defined recursively by $a_1 = 3$, $a_{n+1} = \sqrt{6a_n - 5}$ is increasing.

   (a) Explain briefly what else you need to know to guarantee that the sequence converges and why.

   (b) Assuming your answer to (a) is true, find the limit.