Week 14 Week in Review

courtesy: David J. Manuel

(covering 10.1, 10.2, and 10.3)

1 Section 10.1

1. Given the curve parametrized by $x = \sin(t)$, $y = t^3$, find parametric equations for the line tangent to the curve at $t = \pi$.
2. Find the intersection of the lines parametrized by $x = 3 + 2t$, $y = 2 + t$ and $x = 1 - s$, $y = 1 + 3s$. Are the lines parallel, perpendicular, or neither? Justify your answers WITHOUT finding the Cartesian equations of the lines.
3. Find the horizontal and vertical tangent lines to the curve parametrized by $x = t^2 - 4t$, $y = t^2 - 6t$.

2 Section 10.2

1. Find the length of the curve parametrized by $x = (\sin(t))^2$, $y = (\cos(t))^2$, $t \in \left[0, \frac{\pi}{3}\right]$.
2. Find the length of the curve parametrized by $x = \frac{\sqrt{2}}{3}t^{3/2}$, $y = t + 27$, $t \in [0, 6]$
3. Find the length of the curve parametrized by $x = 1 + e^{-t}\cos(t)$, $y = e^{-t}\sin(t))$, $t \in [0, \infty)$
4. Find the area of the surface formed by rotating the curve parametrized by $x = t^2$, $y = t^3$, $t \in [0, 1]$ about the y-axis.
5. Find the area of the surface formed by rotating the curve parametrized by $x = t - \ln(t)$, $y = 4\sqrt{t}$, $t \in [1, 4]$ about the x-axis.

3 Section 10.3

1. Convert the following points between Cartesian and polar coordinates:
   (a) $(0, 2)$ from Cartesian to polar
   (b) $(-2, -2\sqrt{3})$ from Cartesian to polar
   (c) $\left(4, \frac{3\pi}{4}\right)$ from polar to Cartesian
   (d) $\left(-5, \frac{\pi}{2}\right)$ from polar to Cartesian

2. Sketch the graph of the polar curve $r = 1 + \sin(\theta)$

3. Given the polar curve $r = 4\sin(\theta)$:

   (a) Sketch the graph

   (b) Find the Cartesian equation

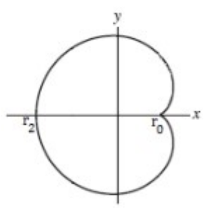
4. Given the Cartesian curve $2(x^2 + y^2)^2 = 25(x^2 - y^2)$

   (a) Find the polar equation of the curve

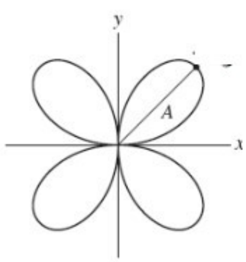
   (b) Sketch the graph

4 Section 10.4

1. Find the area enclosed by the cardioid $r = 9 - \cos(\theta)$.



2. Find the area of one leaf of the four-petal rose $r = 9\sin(2\theta)$.



3. Find the length of the polar curve $r = 11\theta^2$ for $\theta \in [0, \pi]$.

4. Find the length of the spiral $r = e^\theta$ for $\theta \in [0, 2\pi]$.