Instructor: Rosanna Pearlstein

Math 152 – Fall 2022
"Week-in-Review"

Exam2 WIR

$\sec^2\theta = 1 + \tan^2\theta$

① $a^2 - x^2$ $\quad a\sin\theta$
③ $x^2 - a^2$ $\quad a\sec\theta$
② $x^2 + a^2$ $\quad a\tan\theta$

Problem 1. Compute the integral $\int \frac{dx}{x^2\sqrt{x^2-1}}$.

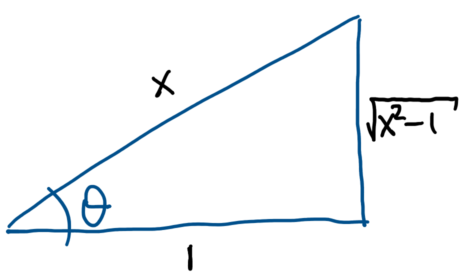
$x = \sec\theta$

$dx = \sec\theta\tan\theta\, d\theta$

$x^2 - 1 = \sec^2\theta - 1 = \tan^2\theta$

$$\int \frac{\cancel{\sec\theta}\,\cancel{\tan\theta}\,d\theta}{\sec^2\theta\,\cancel{\sqrt{\tan^2\theta}}} = \int \frac{1}{\sec\theta}\,d\theta = \int \cos\theta\, d\theta =$$

$$= \sin\theta + C = \frac{\sqrt{x^2-1}}{x} + C.$$



$\sec\theta = \frac{x}{1} \Rightarrow \cos\theta = \frac{1}{x}$

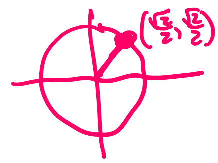
$\sin\theta = \frac{\sqrt{x^2-1}}{x} + C$

Problem 2. Compute the integral $\int_0^2 x^3\sqrt{x^2+4}\,dx$.

If given $\sqrt{9x^2+4}$
then $x = \frac{2}{3}\sec\theta$

$$\boxed{x = 2\tan\theta}$$

$0 = 2\tan\theta \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0$
$2 = 2\tan\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$



$$dx = 2\sec^2\theta\, d\theta$$

$$\sqrt{x^2+4} = \sqrt{4\tan^2\theta + 4} =$$

$$= \sqrt{4\sec^2\theta} = 2\sec\theta$$

$$\int_0^{\pi/4} 8\tan^3\theta \; 2\sec\theta \; 2\sec^2\theta\, d\theta$$

$$32\int_0^{\pi/4} \tan^3\theta \sec^3\theta\, d\theta$$

$$\frac{\sin^3}{\cos^3}\,\frac{1}{\cos^3} = \frac{\sin^3}{\cos^6}$$

$$32\int_0^{\pi/4} \frac{\sin^3\theta}{\cos^6\theta}\, d\theta = 32\int_0^{\pi/4} \frac{\sin^2\theta}{\cos^6\theta}\,\sin\theta\, d\theta$$

$$32\int_0^{\pi/4} \frac{(1-\cos^2\theta)}{\cos^6\theta}\,\sin\theta\, d\theta$$

$u = \cos\theta$ $\quad u = \cos 0 = 1$
$\quad u = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$
$du = -\sin\theta\, d\theta$

$$-32\int_1^{\sqrt{2}/2} \frac{1-u^2}{u^6}\, du = -32\int_1^{\sqrt{2}/2}\left(\frac{1}{u^6} - \frac{u^2}{u^{6}4}\right) du =$$

u^{-6} $\quad u^{-4}$

$$\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\frac{1}{2\sqrt{2}\,2\sqrt{2}\,2\sqrt{2}} = \frac{1}{4\sqrt{2}}$$

$$= -32\left[\frac{u^{-5}}{-5} - \frac{u^{-3}}{-3}\right]_1^{\sqrt{2}/2} = +32\left[\frac{-1}{5u^5} + \frac{1}{3u^3}\right]_{\sqrt{2}/2}^{1} =$$

$$=32\left[\left(-\frac{1}{5}+\frac{1}{3}\right)-\left(-\frac{1}{5}\cdot 4\sqrt{2}+\frac{1}{3}2\sqrt{2}\right)\right]=32\left[\frac{-3+5}{15}+\frac{4}{5}\sqrt{2}-\frac{2}{3}\sqrt{2}\right]=\frac{64}{15}(1+\sqrt{2}).$$

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$1-x^2$ $\quad x=\sin\theta$

Problem 3. Compute the integral $\int \sqrt{-x^2+6x+7}\,dx$.

$$-\left[x^2-6x+9-9-7\right]=-\left[(x-3)^2-16\right]=16-(x-3)^2$$

$$x-3=4\sin\theta \Rightarrow dx=4\cos\theta\, d\theta$$

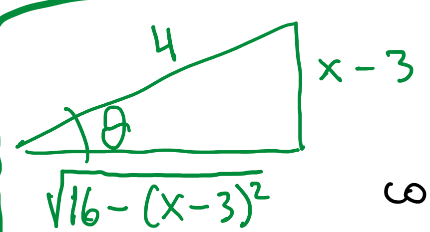
$$\sqrt{16-16\sin^2\theta}=\sqrt{16(1-\sin^2\theta)}=\sqrt{16\cos^2\theta}$$

$$\int\sqrt{16-(x-3)^2}\,dx=\int 4\cos\theta\, 4\cos\theta\, d\theta=$$

$$=16\int\cos^2\theta\, d\theta=\cancel{16}^{\,8}\int\frac{1}{\cancel{2}}(1+\cos 2\theta)\,d\theta=$$

$$\sin\theta=\frac{x-3}{4}$$

$$=8\left[\theta+\frac{1}{2}\sin 2\theta\right]+C=$$



$$=8\left[\theta+\frac{1}{\cancel{2}}\cancel{2}\sin\theta\cos\theta\right]+C$$

$$\cos\theta=\frac{\sqrt{-x^2+6x+7}}{4}$$

$$=8\left[\arcsin\frac{x-3}{4}+\left(\frac{x-3}{4}\right)\left(\frac{\sqrt{-x^2+6x+7}}{4}\right)\right]+C$$

$x^2(x+1)\overline{)x^3+1}$

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Problem 4. Compute the integral $\int_2^3 \frac{x^3+1}{x^2(x+1)}\,dx =$

$$\begin{array}{r} x^2 - x + 1 \\ x+1 \overline{) x^3 + 0x^2 + 0x + 1} \\ -(x^3 + x^2) \\ \hline -x^2 + 0x + 1 \\ -(-x^2 - x) \\ \hline +x + 1 \\ x + 1 \\ \hline 0 \end{array}$$

$$= \int_2^3 \frac{\cancel{(x+1)}(x^2-x+1)}{x^2\cancel{(x+1)}}\,dx =$$

$$= \int_2^3 \frac{x^2-x+1}{x^2}\,dx =$$

$$= \int_2^3 \left(\frac{x^2}{x^2} - \frac{x}{x^2} + \frac{1}{x^2}\right) dx = \int_2^3 \left(1 - \frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$\left[x - \ln|x| - \frac{1}{x}\right]_2^3 =$$

$$3 - \ln 3 - \frac{1}{3} - \left(2 - \ln 2 - \frac{1}{2}\right)$$

$$1 - \frac{1}{3} + \frac{1}{2} - \ln 3 + \ln 2 = \frac{7}{6} + \ln\frac{2}{3}.$$

Problem 5. Compute the integral $\int \frac{x+1}{x^2-4}dx$

$$\frac{x+1}{(x+2)(x-2)} = \frac{A}{x+2}\frac{(x-2)}{x-2} + \frac{B}{x-2}\frac{(x+2)}{x+2}$$

$$x+1 = A(x-2) + B(x+2)$$

$$1 = A + B \Rightarrow A = 1-B$$

$$1 = -2A + 2B = -2(1-B) + 2B = -2 + 2B + 2B$$

$$1 = -2 + 4B \Rightarrow 3 = 4B \Rightarrow B = \frac{3}{4}$$

$$A = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\int \left(\frac{1/4}{x+2} + \frac{3/4}{x-2}\right)dx =$$

$$= \frac{1}{4}\ln|x+2| + \frac{3}{4}\ln|x-2| + C.$$

Problem 6. Compute the integral $\int \frac{2x^2-x+4}{x^3+4x}\,dx$

$$\frac{2x^2-x+4}{x(x^2+4)} = \frac{A}{x}\,\frac{x^2+4}{x^2+4} + \frac{Bx+C}{x^2+4}\,\frac{x}{x}$$

irreducible

$$2x^2 - x + 4 = A(x^2+4) + (Bx+C)x$$

$$2 = A + B \Rightarrow B = 1$$

$$-1 = C \qquad C = -1$$

$$4 = 4A \Rightarrow A = 1$$

$$\int \left(\frac{1}{x} + \frac{x-1}{x^2+4}\right) dx = \int \left(\frac{1}{x} + \frac{x}{x^2+4} - \frac{1}{x^2+4}\right) dx$$

$u = x^2+4$
$du = 2x\,dx \ldots$

$$\ln|x| + \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C.$$

Problem 7. Compute the integral $\int_e^\infty \frac{dx}{x(\ln x)^2}$.

$u = \ln x \qquad du = \frac{1}{x}dx$

$$\int_{\ln e}^{\text{"}\ln\infty\text{"}} \frac{du}{u^2} = -\frac{1}{u}\Big|_1^\infty = -\left(\cancel{\frac{1}{\infty}}^{\,0} - \frac{1}{1}\right) = 1$$

Recall
$\int_0^a \frac{1}{x^p}dx$ convergent when $p<1$

Problem 8. Compute the integral $\int_1^9 \frac{1}{\sqrt[3]{x-9}}dx$.

$u = x-9$
$du = dx$

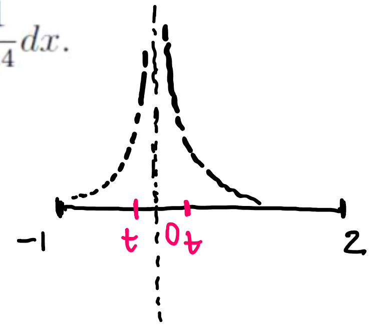
$\int_{1-9}^{9-9} \frac{1}{u^{1/3}}\,du =$

$\int_{-8}^{0} u^{-1/3}\,du = \frac{3}{2}u^{2/3}\Big|_{-8}^{0} = \frac{3}{2}\left(0^{2/3} - (-8)^{2/3}\right) =$

$\frac{3}{2}(-4) = -6$

Recall $\int_0^a \frac{1}{x^p}\,dx$ diverges when $p>1$

Problem 9. Compute the integral $\int_{-1}^{2} \frac{1}{x^4}\,dx$.



$$\int x^{-4}\,dx = \frac{x^{-3}}{-3}\Big|_{-1}^{2}$$

$$\lim_{t\to 0^-}\int_{-1}^{t}\frac{1}{x^4}\,dx + \lim_{t\to 0^+}\int_{t}^{2}\frac{1}{x^4}\,dx =$$

$$= \lim_{t\to 0^-}\left[-\frac{1}{3x^3}\right]_{-1}^{t} + \lim_{t\to 0^+}\left[-\frac{1}{3x^3}\right]_{t}^{2}$$

$$\lim_{t\to 0^-} -\frac{1}{3}\left(\cancel{\frac{1}{t^3}}\to\infty - \frac{1}{-3}\right) + \lim_{t\to 0^+} -\frac{1}{3}\left(\frac{1}{8} - \cancel{\frac{1}{t^3}}\right)\to\infty$$

The integral is divergent.

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Problem 10. Determine whether the integral converges: $\int_1^\infty \frac{1}{x+e^{2x}}\,dx$.

Claim: $\underbrace{\int^\infty \frac{1}{e^{2x}}\,dx}_{1}$ converges

$\int e^{-2x}\,dx = -\frac{1}{2}e^{-2x}$

$-\frac{1}{2}e^{-2x}\Big|_1^\infty = -\frac{1}{2}\left(\cancel{e^{-\infty}}^{\,0} - e^{-2}\right) = \frac{1}{2e^2}$ ✓

$x + e^{2x} > e^{2x}$

$\frac{1}{x+e^{2x}} < \frac{1}{e^{2x}}$

so convergent because less than a convergent integral.

COMPARISON TEST.

Problem 11. Determine whether the integral converges: $\int_5^\infty \frac{x}{x^{3/2}-x-1}dx$.

When x is large $\frac{x}{x^{3/2}-x-1} \approx \frac{x}{x^{3/2}} = x^{1-3/2} = x^{-1/2}$

Given Integral $\approx \int_5^\infty \frac{1}{\sqrt{x}}dx$ diverges

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Problem 12. Determine whether the sequence converges (if it does, find the limit) $a_n = \ln(3n+1) - \ln(4n^2)$.

$$a_n = \underbrace{\ln(3n+1)}_{\infty} - \underbrace{\ln(4n^2)}_{\infty}$$

$\infty - \infty$ indet. form

$$= \ln\left|\frac{3n+1}{4n^2}\right|$$

continuous

$$\lim_{n} a_n = \lim_{n} \ln\left|\frac{3n+1}{4n^2}\right| =$$

$$= \ln\left(\lim_{n\to\infty}\left|\frac{3n+1}{4n^2}\right|\right) = \ln\left(\lim\left|\frac{3n}{4n^2}\right|\right) =$$

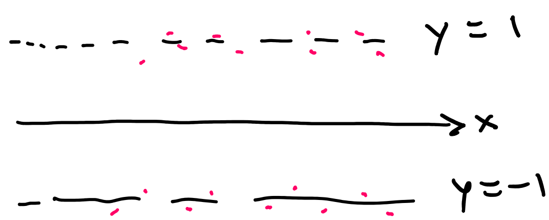
$$= \text{"} \ln 0 \text{" "="} -\infty$$

The integral diverges.

Problem 13. Determine whether the sequence converges (if it does, find the limit) $a_n = (-1)^n \frac{n}{n+1}$.

$$\lim_{n \to \infty} \frac{n}{n+1} = 1$$

$$a_n = (-1)^n \frac{n}{n+1} \to 1$$



Limit DNE does not exist.

Problem 14. Determine whether the sequence converges (if it does, find the limit) $a_n = (-1)^n \frac{n}{n^2+1}$.

$$\lim_{n\to\infty} \frac{n}{n^2+1} = 0$$

$$a_n = (-1)^n \frac{n}{n^2+1}$$

$$-\left(\frac{n}{n^2+1}\right) \le (-1)^n \frac{n}{n^2+1} \le \frac{n}{n^2+1}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$0 \qquad\qquad 0 \qquad\qquad 0$$

SQUEEZE Theorem

Problem 15. Determine whether the sequence converges (if it does, find the limit) $a_n = \sqrt{n^2 - 8n} - n$.

$\to \infty$ $\to \infty$ $\infty - \infty$ ind. form.

$$a_n = \sqrt{n^2-8n} - n \cdot \frac{\sqrt{n^2-8n}+n}{\sqrt{n^2-8n}+n} =$$

$$= \frac{\cancel{n^2} - 8n - \cancel{n^2}}{\sqrt{n^2-8n}+n}$$

$$\lim_{n\to\infty} \frac{-8n}{\sqrt{n^2-8n}+n} = \lim \frac{n(-8)}{\sqrt{n^2\left(1-\frac{8n}{n^2}\right)}+n} =$$

$$= \lim \frac{n(-8)}{n\sqrt{1-\frac{8}{n}}+n} = \lim \frac{\cancel{n}(-8)}{\cancel{n}\left(\sqrt{1-\cancel{\frac{8}{n}}}+1\right)} = \frac{-8}{2} = \boxed{-4}$$

0

$$\sqrt{n^2(\ )} = \sqrt{n^2}\sqrt{(\ )} = n\sqrt{(\ )}$$

Problem 16. Consider the recursive sequence defined by $a_1 = 2$, $a_{n+1} = 1 - \frac{1}{a_n}$. Find the first 5 terms of the sequence. Find the limit of the sequence, if it exists.

$a_1 = 2 \quad a_2 = 1 - \frac{1}{2} = \frac{1}{2} \quad a_3 = 1 - 2 = -1$

$a_4 = 1 - \frac{1}{-1} = 2 \quad a_5 = 1 - \frac{1}{2} = \frac{1}{2}$

$2, \frac{1}{2}, -1, 2, \frac{1}{2}, -1, 2, \frac{1}{2}, -1, 2, \ldots$

NO LIMIT!

Problem 17. Given the recursive sequence below is increasing and bounded, find the limit.
$a_1 = 2$, $a_{n+1} = 4 - \dfrac{3}{a_n}$.

convergent.

If $a_n \to \ell$ then $a_{n+1} \to \ell$
$a_{n-1} \to \ell$

$$\left[\ell = 4 - \frac{3}{\ell}\right](\ell)$$

$$\ell^2 = 4\ell - 3$$

$$\ell^2 - 4\ell + 3 = 0$$

$$(\ell - 3)(\ell - 1) = 0$$

$\ell = \cancel{1}$, 3

Limit $a_n = 3$.

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Problem 18. Use the Test For Divergence to show the series diverges:

$$\sum_{n=1}^{\infty} \frac{n^2}{3(n+1)(n+2)}$$

If $a_n \not\to 0$ as $n \to \infty$, then Σa_n diverges.

Explain why the Test for Divergence is inconclusive when applied to the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$.

yes, $\lim_{n} a_n = \lim \sin \frac{1}{n} = \sin 0 = 0$ so the series can converge or diverge

$$\lim_{n} \frac{n^2}{3(n+1)(n+2)} = \frac{1}{3}$$ (L'HR or algebra...)

$\frac{1}{3} \neq 0$ so the series diverges.

L.

Problem 19. Find the sum of the series: $\sum_{n=1}^{\infty}\left(\sin\frac{1}{n}-\sin\frac{1}{n+1}\right)=$

$\left(\sin 1-\cancel{\sin\frac{1}{2}}\right)+\left(\cancel{\sin\frac{1}{2}}-\cancel{\sin\frac{1}{3}}\right)+\left(\cancel{\sin\frac{1}{3}}-\sin\frac{1}{4}\right)=a_1+a_2+a_3=S_3$

$S_n=\sin 1-\sin\left(\frac{1}{n+1}\right)$

$\lim S_n=\sin 1-\sin 0=\boxed{\sin 1}$

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Problem 20. Find the sum of the series: $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$.

$$\frac{1}{n(n+2)} = \frac{A}{n}\frac{n+2}{n+2} + \frac{B}{n+2}\frac{n}{n}$$

$$0n + 1 = A(n+2) + Bn$$

$$1 = 2A \Rightarrow A = 1/2$$

$$0 = A + B \Rightarrow B = -1/2$$

$$\sum_{n=1}^{\infty} \frac{\frac{1}{2}}{n} - \frac{\frac{1}{2}}{n+2}$$

$$\sum \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$$

$$\frac{1}{2}\left[\left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{5}} - \frac{1}{7}\right) + \left(\cancel{\frac{1}{6}} - \frac{1}{8}\right) \cdots\right]$$

$$S_n = \frac{1}{2}\left[1 + \frac{1}{2} - \underbrace{\cancel{\frac{1}{n+1}}}_{\to 0} - \underbrace{\cancel{\frac{1}{n+2}}}_{\to 0}\right]$$

$$S = \frac{1}{2}\left(1 + \frac{1}{2}\right)$$

Problem 21. Find the sum of the series: $\sum_{n=1}^{\infty} 2\left(\frac{5}{7}\right)^{n-1}$.

$$\underbrace{2\left[1+\frac{5}{7}+\left(\frac{5}{7}\right)^2+\cdots\right]}$$

Geometric series with ratio $r=\frac{5}{7}<1$

$$2\cdot\frac{1}{1-\frac{5}{7}}$$

$$2\cdot\frac{7}{7-5}=\boxed{7}$$

Problem 22. Find the sume of the series: $\sum_{n=1}^{\infty} \frac{3^{2n+1}}{10^n}$.

$$\frac{3^{2+1}}{10} + \frac{3^{4+1}}{10^2} + \frac{3^{6+1}}{10^3} + \cdots$$

$$\frac{3^3}{10}\Big(\underbrace{1 + \frac{3^2}{10} + \frac{(3^2)^2}{10^2} + \cdots}_{\text{Geo series } r=\frac{9}{10}<1}\Big)$$

$$\frac{3^3}{10}\,\frac{1}{1-\frac{9}{10}} =$$

$$= \frac{27}{10}\,\frac{10}{10-9} = \boxed{27}$$

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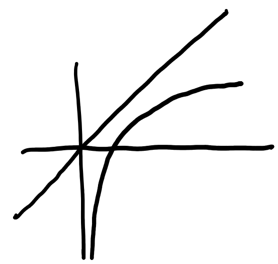
Problem 23. Determine whether the following series converges or diverges: $\sum_{n=2}^{\infty} \frac{(+1)^n}{\ln n}$.

$$\sum_{n=2}^{\infty} \frac{1}{\ln n}$$

Recall: $\sum_{n=2}^{\infty} \frac{1}{n}$ diverges



$\ln n < n \quad (n > 2)$

$\frac{1}{\ln n} > \frac{1}{n}$

series is bigger than a divergent

so it's divergent.

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Problem 24. Determine whether the following series converges or diverges: $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{\sqrt{n+1}}$

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}} \qquad \frac{\sqrt{n}}{\sqrt{n(1+\frac{1}{n})}} = \frac{\cancel{\sqrt{n}}\ 1}{\cancel{\sqrt{n}}\sqrt{1+\cancel{\frac{1}{n}}}} = 1$$

0

since a_n doesn't converge to zero,

the series is divergent

Problem 25. Determine whether the following series converges or diverges: $\sum_{n=2}^{\infty} \frac{(+1)^n}{3n-1}$.

Integral test:

$$\int_2^{\infty} \frac{1}{3x-1}\,dx = \ln|3x-1|\Big|_2^{\infty} =$$

$$= \underbrace{\ln|3\cdot\infty - 1|}_{\infty} - \ln 2$$

$$\infty - \ln 2 = \infty$$

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Problem 26. Determine whether the following series converges or diverges: $\sum_{n=1}^{\infty} \frac{(+1)^n n}{n^3+1}$.

We will learn how to solve this by Limit Comparison Test

not on exam.

Problem 27. Determine whether the following series converges or diverges: $\sum_{n=1}^{\infty} \frac{\cos\left(\frac{1}{n}\right)}{n^2}$.

not on exam.

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Problem 28. Determine whether the following series converges or diverges: $\sum_{n=1}^{\infty} \frac{(-10)^n n!}{(2n+1)!}$

will learn this next week

not on exam.

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In the problem below, the "-1" is a typo, we will solve replacing "-1" by "+1".

Problem 29. Consider the series $\sum_{n=1}^{\infty} \frac{(+1)^n}{n^5}$. Use the first 5 terms to estimate the sum. Estimate the error in the approximation s_5 to the sum of the series. How many terms do you need to take in order to ensure an approximation to within .01?

$$1 + \frac{1}{2^5} + \frac{1}{3^5} + \frac{1}{4^5} + \frac{1}{5^5} = \ldots$$

$$\int_6^\infty \frac{1}{x^5}dx \le R_5 \le \int_5^\infty \frac{1}{x^5}dx =$$

$$\int x^{-5} = \frac{x^{-4}}{-4}$$

$$= -\frac{1}{4}\frac{1}{x^4}\Big|_5^\infty = \boxed{-\frac{1}{4}\left(\frac{1}{\infty^4} - \frac{1}{5^4}\right)} = \frac{1}{4\cdot 5^4} = .0005$$

want: $R_n < \frac{1}{100}$

$$\int_n^\infty \frac{1}{x^5}dx = \frac{1}{4(n^5)} < \frac{1}{100}$$

$$4(n^5) > 100$$

$$n^5 > \frac{100}{4} = 25$$

$$\boxed{\text{take } n = 2}$$

$$2^5 = 32 > 25$$