Week 4

Monday, February 13, 2023 5:55 PM

151WIRH4

MATH 151- WEEK-IN-REVIEW 4

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EXAM 1 REVIEW

1. Evaluate the following limits.

(a) $\displaystyle\lim_{x\to\infty} \frac{-10x^3 + 7x - 15}{3x^2} = \lim_{x\to\infty} \frac{-10x^3}{3x^2} = \lim_{x\to\infty} \frac{-10x}{3} = \boxed{-\infty}$

(b) $\displaystyle\lim_{x\to-\infty} \frac{x-7}{3+2x^2} = \lim_{x\to-\infty} \frac{x}{2x^2} = \lim_{x\to-\infty} \frac{1}{2x} = \boxed{0}$

(c) $\displaystyle\lim_{x\to-\infty} \frac{x^3 - 2x}{3 - 2x^2 - 5x^3} = \lim_{x\to-\infty} \frac{x^3}{-5x^3} = \boxed{-\frac{1}{5}}$

(d) $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{4x^6 \cancel{+7x} \cancel{-1}}}{x^3 \cancel{+9}} = \lim_{x\to-\infty} \frac{\sqrt{4x^6}}{x^3} = \lim_{x\to-\infty} \frac{2|x^3|}{x^3} = 2(-1) = -2$

OR $\displaystyle = \lim_{x\to-\infty} \frac{\sqrt{4x^6}}{-\sqrt{x^6}} = \lim_{x\to-\infty} -\sqrt{\frac{4x^6}{x^6}} = -\sqrt{4} = -2$

(e) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{4x^6 + 7x - 1}}{x^3 + 9} = \lim_{x\to\infty} \frac{\sqrt{4x^6}}{x^3} = \lim_{x\to\infty} \frac{2|x^3|}{x^3} = 2(1) = 2$

(f) $\lim\limits_{x\to\infty} \dfrac{(\sqrt{4x^2+7x+3}-2x)}{1} \cdot \dfrac{(\sqrt{4x^2+7x+3}+2x)}{(\sqrt{4x^2+7x+3}+2x)}$

$$= \lim_{x\to\infty} \frac{(\cancel{4x^2}+7x+3-\cancel{4x^2})}{\sqrt{4x^2+7x+3}+2x} = \lim_{x\to\infty} \frac{(7x+3)/x}{(\underbrace{\sqrt{4x^2+7x+3}}_{2x'}+2x)/x}$$

$$\frac{\sqrt{4x^2+7x+3}}{x} = \sqrt{\frac{4x^2+7x+3}{x^2}}$$

$$= \lim_{x\to\infty} \frac{7+\cancel{\frac{3}{x}}^{\,0}}{\sqrt{4+\cancel{\frac{7}{x}}^{\,0}+\cancel{\frac{3}{x^2}}^{\,0}}+2}$$

$$= \frac{7}{\sqrt{4}+2} = \boxed{\frac{7}{4}}$$

(g) $\lim\limits_{x\to-\infty} (\sqrt{4x^2+7x+3}-2x)$ $= \ldots = \lim\limits_{x\to-\infty} \dfrac{(7x+3)/x}{(\sqrt{4x^2+7x+3}+2x)/x}$

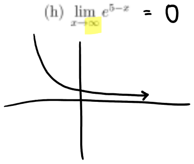
$$\frac{\sqrt{4x^2+7x+3}}{\to x} = -\sqrt{\frac{4x^2+7x+3}{x^2}}$$

$$= \lim_{x\to-\infty} \frac{7+\cancel{\frac{3}{x}}^{\,0}}{-\underbrace{\sqrt{4+\frac{7}{x}+\frac{3}{x^2}}}_{<2}+2} = \frac{7}{\text{tiny positive}} = \boxed{\infty}$$

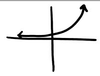
tiny ~~negative~~

$-1.9+2$ tiny positive

(h) $\lim\limits_{x\to\infty} e^{5-x} = 0$



(i) $\lim_{x\to 5^+} e^{\frac{x}{5-x}}$



$\lim_{x\to 5^+}\left(\frac{x}{5-x}\right) = \frac{5}{\text{tiny neg}} = -\infty$

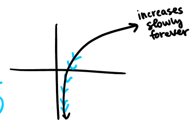
$\hookrightarrow \lim_{x\to -\infty} e^x = 0$

(j) $\lim_{x\to\infty}(\ln(3x^2+5)-\ln(7+6x^2))$

$= \lim_{x\to\infty} \ln\left(\frac{3x^2+5}{7+6x^2}\right) = \boxed{\ln\left(\frac{3}{6}\right) = \ln\left(\frac{1}{2}\right)}$

(k) $\lim_{x\to\infty}(\ln(3x^2+5)-\ln(7+6x)) = \lim_{x\to\infty} \ln\left(\frac{3x^2+5}{7+6x}\right)$

$= \lim_{x\to\infty} \ln\left(\frac{1}{2}x\right) = \boxed{\infty}$



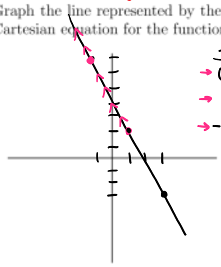
$\lim_{x\to\infty} \ln\left(\frac{3x+5}{7+6x^2}\right) = \lim_{x\to 0^+} \ln(x)$

 0 $\qquad = -\infty$

2. Graph the line represented by the parametric equations $x = 1 - 2t$, $y = 5t + 2$. Then write the Cartesian equation for the function.

$x_0 + m_x t \qquad y_0 + m_y t$

$m_x = -2$ left $\qquad$ up



t	x	y
0	1	2
1	-1	7
-1	3	-3

$y = 5t + 2$

$y - 2 = 5t$

$t = \frac{y-2}{5}$

$x = 1 - 2t$

$\boxed{x = 1 - 2\left(\frac{y-2}{5}\right)}$

$y = 5\left(\frac{1-x}{2}\right) + 2$

3. Find the work done by a force (in Newtons) of $\mathbf{F} = \langle 2, 5\rangle$ and moving an object 6 meters due east.

$\langle 6,0\rangle$

$$\vec{F}\cdot\vec{D} = \langle 2,5\rangle \cdot \langle 6,0\rangle$$
$$= 12 + 0$$
$$= 12\ J$$

$m_x > 0$
$\Rightarrow$ Right

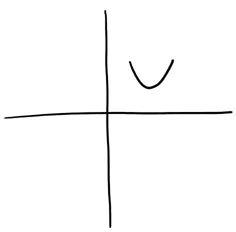
4. Consider the curve $x(t) = t - 2$, $y(t) = t^2 - 3$. (a) Is the point $(4, 40)$ on the graph of the curve? (b) Eliminate the parameter to find a Cartesian equation.

(a) $4 = t-2$
$t = 6$ → $y = (6)^2 - 3$
$= 33$

(4,40) is NOT on the curve

$x = t-2$
$x+2 = t$ → $\boxed{y = (x+2)^2 - 3}$

5. Describe the shape of the curve given by $(2 + \cos(t))\mathbf{i} + (2 - \sin^2(t))\mathbf{j}$.



$x = 2+\cos(t)$
$(x-2)^2 = (\cos(t))^2$
$(x-2)^2 = \cos^2(t)$

$y = 2 - \sin^2(t)$
$\sin^2(t) = 2-y$

$\sin^2(t) + \cos^2(t) = 1$

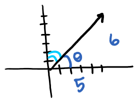
$x = (y-2)^2 + 1$
Right facing

$2 - y + (x-2)^2 = 1$
$y = (x-2)^2 + 2 - 1$
$y = (x-2)^2 + 1$

(partial)
upward facing
parabola

6. Two forces simultaneously act on an object sitting at the origin. The forces are given by $\mathbf{F}_1 = \langle 2, -1\rangle$ and $\mathbf{F}_2 = \langle 3, 7\rangle$. Find the magnitude and direction of the resultant vector. Measure the direction from the positive x-axis.

$\vec{F} = \langle 5, 6\rangle$



$\boxed{\arctan\left(\frac{6}{5}\right) = \theta}$

$\|\vec{F}\| = \sqrt{(5)^2 + (6)^2} = \boxed{\sqrt{61}}$

7. Find the vector projection of $\mathbf{b} = \langle -3, -2\rangle$ onto $\mathbf{a} = \langle -1, 5\rangle$

$\text{proj}_a \vec{b} = \frac{\vec{b}\cdot\vec{a}}{\|\vec{a}\|} \cdot \frac{\vec{a}}{\|\vec{a}\|}$ OR $\frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|^2}\vec{a}$

$= \frac{-7}{\sqrt{26}} \cdot \frac{\langle -1, 5\rangle}{\sqrt{26}} = \frac{\langle 7, -35\rangle}{26} = \boxed{\left\langle \frac{7}{26}, \frac{-35}{26}\right\rangle}$

8. Find the distance from the point $(1, -4)$ to the line $3x + 2y = -2$.

(slope of line)

$\vec{a} = \langle 2, -3\rangle$ $\quad \vec{b} = \langle 1, -3\rangle$

$\vec{a}_\perp = \langle 3, 2\rangle$ $\quad$ (line → point)

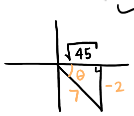
$2y = -3x - 2$

$y = \boxed{-\frac{3}{2}}x - 1$ ← y-int

$(0, -1)$

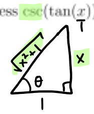
$\frac{\vec{a}_\perp \cdot \vec{b}}{\|\vec{a}_\perp\|} = \frac{3-6}{\sqrt{13}} = \frac{-3}{\sqrt{13}}$ $\quad \boxed{\text{distance} = \frac{3}{\sqrt{13}}}$

9. Evaluate $\cos\left(\underbrace{\arcsin\left(-\frac{2}{7}\right)}\right)$



$\cos(\theta) = \boxed{\frac{\sqrt{45}}{7}}$

10. Express csc(tan(x)) as an algebraic expression.



$$\csc(\theta) = \frac{\sqrt{x^2+1}}{x}$$

11. Find the vertical and horizontal asymptotes for the curve $f(x) = \dfrac{\sqrt{9x^6+1}}{x^3 - 4x^2 - 3x + 12}$.

$$\lim_{x\to\infty} \frac{\sqrt{9x^6 + \cancel{1}}}{x^3 - \cancel{4x^2} - \cancel{3x} + \cancel{12}} = \lim_{x\to\infty} \frac{\sqrt{9x^6}}{x^3} = \lim_{x\to\infty} \frac{3|\cancel{x^3}|}{\cancel{x^3}} = \boxed{3}$$

$$= \frac{\sqrt{9x^6+1}}{x^2(x-4)-3(x-4)}$$

$$\lim_{x\to-\infty} \frac{\sqrt{9x^6+1}}{x^3-4x^2-3x+12} = \lim_{x\to-\infty} \frac{\frac{\sqrt{9x^6+1}}{x^3}}{1-\frac{4}{x}-\frac{3}{x^2}+\frac{12}{x^3}} = \lim_{x\to-\infty} \frac{-\sqrt{\frac{9x^6+1}{x^6}}}{1-\frac{4}{x}-\frac{3}{x^2}+\frac{12}{x^3}} = \frac{-\sqrt{9}}{1} = \boxed{-3}$$

$$= \frac{\sqrt{9x^6+1}}{(x-4)(x^2-3)}$$

VA: $x = 4, \pm\sqrt{3}$

12. Use the limit definition to find the derivative, $f'(x)$, for $f(x) = \sqrt{3-2x}$. No points will be awarded for not using the limit definition of the derivative.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to0} \frac{\left(\sqrt{3-2(x+h)} - \sqrt{3-2x}\right)}{h} \cdot \frac{\left(\sqrt{3-2(x+h)} + \sqrt{3-2x}\right)}{\left(\sqrt{3-2(x+h)} + \sqrt{3-2x}\right)}$$

$$= \lim_{h\to0} \frac{(\cancel{3}-2(x+h)) - (\cancel{3}-\cancel{2x})}{h\left(\sqrt{3-2(x+h)} + \sqrt{3-2x}\right)} = \lim_{h\to0} \frac{-2\cancel{h}}{\cancel{h}\left(\sqrt{3-2(x+h)} + \sqrt{3-2x}\right)}$$

$$= \lim_{h\to0} \frac{-2}{\sqrt{3-2(x+\cancel{h})} + \sqrt{3-2x}} = \frac{-2}{\sqrt{3-2x}+\sqrt{3-2x}} = \frac{-\cancel{2}}{\cancel{2}\sqrt{3-2x}} = \boxed{\frac{-1}{\sqrt{3-2x}}}$$

$f'(x)$ = slope of tangent line
= Instantaneous rate of change

Find the equation of the tangent line for the above function at $x = -1$.

$$y - y_1 = m(x - x_1)$$

$(-1, \sqrt{5})$ $\quad f(-1) = \sqrt{3-2(-1)} = \sqrt{5}$

$$m = \frac{-1}{\sqrt{3-2(-1)}} = -\frac{1}{\sqrt{5}}$$

$$\boxed{y - \sqrt{5} = -\frac{1}{\sqrt{5}}(x+1)}$$

Slope of $\frac{5}{x+2}$ at $x = 3$

$$\lim_{x\to3} \frac{f(x)-f(3)}{x-3} = \lim_{x\to3} \frac{\left(\frac{5}{x+2} - 1\right)(x+2)}{(x-3)(x+2)}$$

$$= \lim_{x\to3} \frac{5-(x+2)}{(x-3)(x+2)}$$

~~5-x-2~~

$$= \lim_{x\to3} \frac{3-x}{(x-3)(x+2)}$$

$$\lim_{x\to3} \frac{-(\cancel{x-3})}{(\cancel{x-3})(x+2)}$$

$$= \boxed{\frac{-1}{5} = m}$$

13. Find the limit or prove it does not exist. Do not use L'Hôpital's Rule.

(a) $\lim\limits_{x\to 2} \dfrac{\left(\dfrac{2}{2x+1}-\dfrac{2}{5}\right)(2x+1)(5)}{(x-2)(2x+1)(5)}$

$= \lim\limits_{x\to 2} \dfrac{(2)(5)-2(2x+1)}{(x-2)(2x+1)(5)} = \lim\limits_{x\to 2} \dfrac{10-4x-2}{(x-2)(2x+1)(5)} = \lim\limits_{x\to 2} \dfrac{8-4x}{(x-2)(2x+1)(5)}$

$= \lim\limits_{x\to 2} \dfrac{-4\cancel{(x-2)}}{\cancel{(x-2)}(2x+1)(5)} = \lim\limits_{x\to 2} \dfrac{-4}{(2x+1)(5)} = \boxed{-\dfrac{4}{25}}$

(b) $\lim\limits_{x\to -3} \dfrac{2x^4-162}{x+3}$

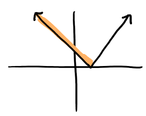
$= \lim\limits_{x\to -3} \dfrac{2(x^4-81)}{(x+3)} = \lim\limits_{x\to -3} \dfrac{2(x^2+9)(x^2-9)}{(x+3)} = \lim\limits_{x\to -3} \dfrac{2(x^2+9)\cancel{(x+3)}(x-3)}{\cancel{(x+3)}}$

$= \lim\limits_{x\to -3} 2(x^2+9)(x-3) = 2(18)(-6) = \boxed{-216}$

(c) $\lim\limits_{x\to -4} \dfrac{3+x}{(x+4)^2} = \lim\limits_{x\to -4} \dfrac{-1}{(\underset{\text{neg/pos}}{\text{tiny}})^2} = \dfrac{-1}{\underline{\text{tiiiny}}\ \text{pos.}} = \boxed{-\infty}$

$(-3.9+4)^2$
$=(0.1)^2$

$(-4.01+4)^2$
$=(-0.01)^2$

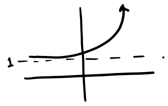


(d) $\lim\limits_{x\to 1^-} \dfrac{x^2+3x-4}{|1-x|} = \lim\limits_{x\to 1^-} \dfrac{(x+4)(x-1)}{|1-x|} = \lim\limits_{x\to 1^-} \dfrac{-(x+4)(1-x)}{|1-x|}$

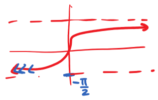
$= \lim\limits_{x\to 1^-} \dfrac{-(x+4)\cancel{(1-x)}}{\cancel{(1-x)}} = \boxed{-5}$

$|1-x| = \begin{cases} 1-x & x<1 \\ -(1-x) & x>1 \\ x-1 & \end{cases}$

(e) $\lim_{x\to-\infty}(e^{2x}+1)$ $= 0+1 = 1$

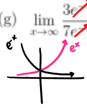


(f) $\lim_{x\to\infty} \arctan\left(\frac{3-4x^3}{1+5x^2}\right)$



$= \lim_{x\to\infty} \arctan\left(\frac{-4x}{5}\right) = \boxed{\frac{-\pi}{2}}$

(g) $\lim_{x\to\infty} \frac{3e^{-5x}+9e^{3x}}{7e^{-5x}-3e^{3x}} = -3$



$\lim_{x\to-\infty} \frac{3e^{-5x}+9e^{3x}}{7e^{-5x}-3e^{3x}} = \frac{3}{7}$

14. Which of following intervals must contain a solution to the equation $\underbrace{2x^3 + 16x + 3}$ = 18?
(a) $[-2, -1]$
(b) $[-1, 0]$
(c) $[0, 1]$ → @0 → $\boxed{3}$ @1 → $\boxed{21}$
(d) $[1, 2]$
(e) $[2, 3]$

15. Given $f(x) = \begin{cases} x^2 - 5a & \text{if } x < -1 \\ ax^2 & \text{if } -1 \le x \le 2 \\ 3ax + b & \text{if } x > 2 \end{cases}$. Find values for a and b that make the function continuous everywhere.

① $\lim_{x \to -1^-} (x^2 - 5a) = 1 - 5a$

$\lim_{x \to -1^+} (ax^2) = a$

$1 - 5a = a$
$1 = 6a$
$\boxed{a = 1/6}$

② $\lim_{x \to 2^-} ax^2 = 4a$

$\lim_{x \to 2^+} (3ax + b) = 6a + b$

$4a = 6a + b$
$-2a = b$
$\boxed{b = -\frac{1}{3}}$

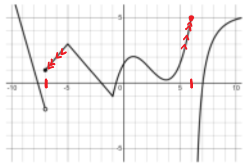
16. Given SQUEEZE $\underbrace{-2x+1} \le f(x) \le \underbrace{3x^2}$, compute $\lim_{x \to -1} f(x)$.

$\lim_{x \to -1} (-2x+1) = 3$

$\lim_{x \to -1} (3x^2) = 3$

$\boxed{\lim_{x \to -1} f(x) = 3}$

17. Determine the following limits given the graph of $f(x)$ to the right.



(a) $\lim_{x\to -7} f(x)$ DNE

(b) $\lim_{x\to -1} f(x) = -1$

(c) $\lim_{x\to 1} f(x) = 2$

(d) $\lim_{x\to 6^-} f(x) = 5$

(e) $\lim_{x\to 6^+} f(x) = -\infty$

(f) $\lim_{x\to 6} f(x)$ DNE

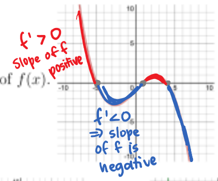
Is there anywhere $f(x)$ is discontinuous? If so, is it left continuous, right continuous, or neither?

$x=-7$ continuous from right only
$x=6$ continuous from left only

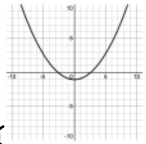
Is there anywhere $f(x)$ is not differentiable?

$x=-7, 6$ because discontinuous
$x=-1, -5$ because cusps
(too sharp!)

18. Given $f'(x)$ to the right, choose the graph of $f(x)$.



$f'(x)=0 \Rightarrow$ slope of tangent $=0 \Rightarrow$ flat

(A)


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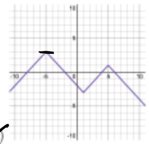

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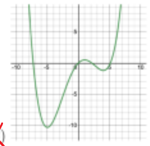

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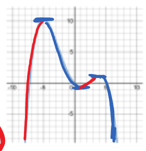

(E)