Week 2

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151WIRH2

Math 151- Week-In-Review 2

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Problem Statements

1. Find the scalar and vector projection of $\langle 3,2\rangle$ onto $\langle -1,5\rangle$.

$\text{Comp}_{\vec a}\vec b = \dfrac{\vec a\cdot\vec b}{\|\vec a\|}$

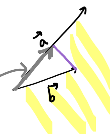
$= \dfrac{3(-1)+2(5)}{\sqrt{26}}$

$= \boxed{\dfrac{7}{\sqrt{26}}}$ length of

$\text{proj}_{\vec a}\vec b = \dfrac{\vec a\cdot\vec b}{\|\vec a\|}\cdot\dfrac{\vec a}{\|\vec a\|}$ $\qquad \dfrac{\vec a\cdot\vec b}{\|\vec a\|^2}\cdot\vec a$

$= \dfrac{7}{\sqrt{26}}\cdot\dfrac{\langle -1,5\rangle}{\sqrt{26}}$

$= \dfrac{7}{26}\langle -1,5\rangle = \boxed{\left\langle \dfrac{-7}{26}, \dfrac{35}{26}\right\rangle}$



2. Find the distance from the point $\boxed{\langle -1,5\rangle}$ to the line $3x+2y=5$.

① $\vec a = \langle 2,-3\rangle$ (slope vector)

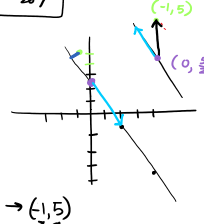
② $\vec b = \boxed{\langle -2,5\rangle}$ (line to point)

③ $\vec a_\perp = \langle 3,2\rangle$ (perpendicular to slope)

④ $\text{Comp}_{\vec a_\perp}\vec b = \dfrac{\vec a_\perp\cdot\vec b}{\|\vec a_\perp\|} = \dfrac{-2\cdot 3+5\cdot 2}{\sqrt{3^2+2^2}} = \boxed{\dfrac{4}{\sqrt{13}}}$

$2y = -3x+5$

$y = -\frac{3}{2}x + \frac{5}{2}$



Vector line to point: $(0,\frac{5}{2}) \rightarrow (-1,5)$

$2\langle -1,\frac{5}{2}\rangle = \langle -2,5\rangle$

$\langle -1-0,\ 5-\frac{5}{2}\rangle$

$\vec a = \langle -1,4\rangle$ & $\vec b\langle 1,6\rangle$

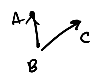
$\vec a\cdot\vec b = \|\vec a\|\cdot\|\vec b\|\cdot\cos\theta$

$\cos\theta = \dfrac{\vec a\cdot\vec b}{\|\vec a\|\cdot\|\vec b\|}$

$\cos\theta = \dfrac{23}{\sqrt{17}\sqrt{37}}$

A(1,1) B(-1,2) C(0,3)

$\measuredangle ABC$



$\vec{BA}\langle 2,-1\rangle$ $\vec{BC} = \langle 1,1\rangle$

$\cos\theta = \dfrac{1}{\sqrt5\sqrt2} = \dfrac{1}{\sqrt{10}}$

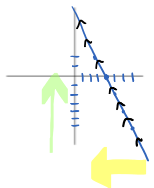
3. Eliminate the parameter to find the Cartesian equation of each curve below. Sketch the parametric curves and indicate the direction in which the curve is traced with an arrow.

(a) $x = 5 - t$, $y = 2t - 2$

$x - 5 = -t$

$t = 5 - x$

$\boxed{y = 2(5-x) - 2}$



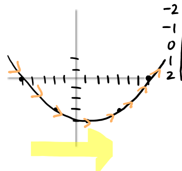
t	x	y
-2	(7	, -6)
-1	6	-4
0	5	-2
1	4	0
2	3	2

(b) $x = 3t + 1$, $y = t^2 - 4$

$x - 1 = 3t$

$\frac{x-1}{3} = t$

$\boxed{y = \left(\frac{x-1}{3}\right)^2 - 4}$

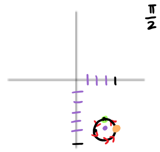


t	x	y
-2	-5	0
-1	-2	-3
0	1	-4
1	4	-3
2	7	0

(c) $x = \cos(\theta) + 3$, $y = \sin(\theta) - 5$, $0 \le \theta \le 2\pi$

$\sin^2\theta + \cos^2\theta = 1$

θ	x	y
0	4	-5
$\frac{\pi}{2}$	3	-4



$x = \cos\theta + 3$

$x - 3 = \cos\theta$

$(x-3)^2 = \boxed{\cos^2\theta}$

$y = \sin\theta - 5$

$y + 5 = \sin\theta$

$\boxed{(y+5)^2 = \sin^2\theta}$

$\boxed{(y+5)^2 + (x-3)^2 = 1}$

Down Right

$\frac{(y+5)^2}{9} + \frac{(x-3)^2}{4} = 1$

up 3
down 3

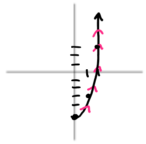
Right 2
Left 2

(a) $\mathbf{r}(t) = \langle \sqrt{t}, 2t-5 \rangle$

$x = \sqrt{t}$ $\quad y = 2t-5$

$\boxed{x = \sqrt{\frac{y+5}{2}}}$ $\quad \frac{y+5}{2} = t$

t	x	y
0	0	−5
1	1	−3
4	2	3



(b) $\mathbf{r}(\theta) = \langle \sin(\theta), \csc^2(\theta) \rangle$ never negative

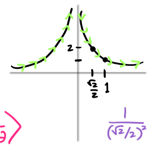
$x^2 = (\sin\theta)^2$

$y = \csc^2\theta$

$\csc^2\theta = \frac{1}{\sin^2\theta}$

$\downarrow$

$y = \frac{1}{x^2}$



θ	x	
$\frac{\pi}{4}$	$\sqrt{2}/2$	2
$\frac{\pi}{2}$	1	1

$\langle \sin\theta, \csc^2\theta \rangle = \langle \sin\theta, \frac{1}{\sin^2\theta} \rangle$

$\frac{1}{(\sqrt{2}/2)^2} = \frac{1}{2/4} = 2$

4. (a) Find a vector equation of the line passing through the points (2, 5) and (−1, 8).

$\boxed{m = \frac{y_2-y_1}{x_2-x_1}} = \frac{3}{-3} = -1 \rightarrow \vec{m} = \boxed{\langle -3, 3 \rangle} = \langle 72, -72 \rangle = \langle 5, 5 \rangle$

$\boxed{x(t) = -3t+2}$ $\quad \boxed{y(t) = 3t+5}$ $\quad \vec{r}(t) = \langle -3t+2, 3t+5 \rangle$

(b) Find a vector passing through the point $\boxed{(2,5)}$ and perpendicular to the line in part (a).

$\vec{m}_\perp = \langle 3, 3 \rangle$

$x(t) = 2 + 3t$

$y(t) = 5 + 3t$

(c) Find a vector that is perpendicular to the line $3x - 7y = 4$.

$\vec{m} = \langle 7, 3 \rangle$

$\vec{m}_\perp = \langle -3, 7 \rangle$

or $\langle 3, -7 \rangle$ or $\langle -6, 14 \rangle$

$3x - 4 = 7y$

$\frac{3}{7}x - \frac{4}{7} = y$

2. Find a vector equation of the line that passes through $(-6, 4)$ parametric equations $x = 7 + 2t$, $y = 1 - 3t$.

~~(a)~~ $\mathbf{r}(t) = \langle 4 + 3t, -6 + 2t \rangle$

(b) $\mathbf{r}(t) = \langle -6 - t, 4 + 7t \rangle \rightarrow m = \frac{7}{-1}$

(c) $\mathbf{r}(t) = \langle -6 + 2t, 4 - 3t \rangle \rightarrow m = \frac{-3}{2}$

~~(d)~~ $\mathbf{r}(t) = \langle 4 + 2t, -6 - 3t \rangle$

(e) $\cancel{\mathbf{r}(t) = \langle -6 + 2t, 4 + 2t \rangle} \rightarrow m = \frac{2}{3}$

$= \boxed{\langle -6 + 9t, 4 + 6t \rangle} \rightarrow m = \frac{6}{9} = \frac{2}{3}$

$m = -\frac{3}{2}$

$m_\perp = \frac{2}{3}$

5. Determine if the following lines are perpendicular, parallel, or neither. If they are not parallel, find the point of intersection.

$L_1 : \langle 5 - 3t, t + 1 \rangle$ → $m = \frac{1}{-3}$

$L_2 : \langle 4s + 1, 12s + 1 \rangle$ → $m = \frac{12}{4} = 3$

$\vec{m}_1 = \langle -3, 1 \rangle$

$\vec{m}_2 = \langle 4, 12 \rangle$

$\vec{m}_1 \cdot \vec{m}_2 = -3 \cdot 4 + 1 \cdot 12$

$= 0 \Rightarrow$ perpendicular

$5 - 3t = 4s + 1$

$(t + 1 = 12s + 1)3$

$3t + 3 = 36s + 3$

$8 = 40s + 4$

$4 = 40s$

$s = \frac{1}{10}$ → $L_2 : \langle 4(\frac{1}{10}) + 1, 12(\frac{1}{10}) + 1 \rangle$

$= \boxed{\left(\frac{7}{5}, \frac{11}{5}\right)}$

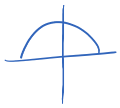
6. Find the exact value of the expression.

(a) $\arctan\left(\frac{\sqrt{3}}{3}\right) = \arctan\left(\frac{1}{\sqrt{3}}\right)$ sin cos

$= \frac{\pi}{6}$

(b) $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

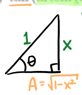
Q2



(c) $\sin\left(2 \cdot \sin^{-1}\left(\frac{3}{4}\right)\right)$

7. Simplify the expression.

(a) $\tan(\arcsin(x))$



$\sin\theta = \frac{x}{1}$

$A^2 + x^2 = 1^2$

$A^2 = 1 - x^2$

$A = \sqrt{1 - x^2}$

$\tan\theta = \boxed{\frac{x}{\sqrt{1 - x^2}}}$

(b) $\sin(\tan^{-1}(x))$

$H^2 = x^2 + 1^2$

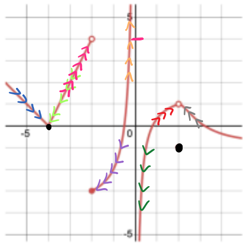
$H = \sqrt{x^2 + 1}$

$\boxed{\sin\theta = \frac{x}{\sqrt{x^2 + 1}}}$

$\tan^{-1}(x) = \theta$

$x = \tan(\theta)$

8. State the value of the given quantity, if it exists, from the given graph of $f(x)$ below.



$\lim_{x\to-4^-} f(x) = 0$ left	$\lim_{x\to-2^-} f(x) = 4$	$\lim_{x\to0^-} f(x) = \infty$	$\lim_{x\to2^-} f(x) = 1$
$\lim_{x\to-4^+} f(x) = 0$ right	$\lim_{x\to-2^+} f(x) = -3$	$\lim_{x\to0^+} f(x) = -\infty$	$\lim_{x\to2^+} f(x) = 1$
$\lim_{x\to-4} f(x) = 0$	$\lim_{x\to-2} f(x)$ DNE	$\lim_{x\to0} f(x)$ DNE	$\lim_{x\to2} f(x) = 1$
$f(-4) = 0$	$f(-2) = -3$	$f(0)$ DNE	$f(2) = -1$