

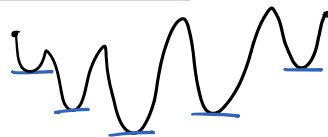
# Week 11

Monday, May 1, 2023 5:51 PM



151WIRH11

MATH 151- WEEK-IN-REVIEW 12  
 ALEXANDRA L. FORAN



FINAL EXAM REVIEW

1. For a continuous function  $f$ , if  $f'(3) = 0$  and  $f''(3) = 7$ , which of these statements do we know to be true about the graph of  $f$  at  $x = 3$ ?

- (a) There is a local maximum at  $x = 3$ .
- (b) There is an absolute maximum at  $x = 3$ .
- (c) There is a local minimum at  $x = 3$ .
- (d) There is an absolute minimum at  $x = 3$ .
- (e) There is not enough information to determine the behavior of the graph at  $x = 3$ .

2. Find the  $x$ -values where local maximums or local minimums occur for  $y = \frac{24}{x^2} + 12x + b$ .

- (a) local min at  $x = \sqrt[3]{4}$  only
- (b) local max at  $x = 0$ , local min at  $x = \sqrt[3]{4}$
- (c) local min at  $x = 0$ , local max at  $x = \sqrt[3]{4}$
- (d) local max at  $x = \sqrt[3]{4}$  only
- (e) local max at  $x = 0$  only

$y' = -\frac{48}{x^2} + 12$

$0 = -\frac{48}{x^2} + 12$   
 $\frac{48}{x^2} = 12$   
 $x^2 = 4$   
 $x = \sqrt[3]{4}$

$y' \text{ DNE @ } x=0$   
 0 is not in the domain  $\Rightarrow$  Not a CV

3. The function  $f(x)$  is defined at all real numbers except 8 and  $f'(x) = \frac{-7(x-1)(x+4)}{(x-8)^4}$ . At what  $x$ -value does  $f(x)$  have a local minimum?

- (a)  $x = 8$  only
- (b)  $x = 1$  only
- (c)  $x = -4$  only
- (d)  $x = 1$  and  $x = 8$  only
- (e)  $f(x)$  does not have a local maximum.



4. Evaluate  $\lim_{x \rightarrow 0^-} \frac{e^{4x} - 5 + 4(x+1)}{x^2}$ .  $\frac{1-5+4}{0} = \frac{0}{0}$

(a) 0  
(b) 8  
(c)  $\infty$   
(d) 2  
(e)  $-\infty$

*Handwritten work:*  

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0^-} \frac{4e^{4x} + 4}{2x} = \frac{8}{0} = \frac{8}{\text{tiny negative}}$$

5. Approximate the area under the curve  $f(x) = x^2 - 1$  on the interval  $[2, 8]$  using three rectangles of equal width and midpoints.

(a) 106  
(b) 226  
(c) 80  
(d) 304  
(e) 160

*Handwritten work:*  
Widths  
 $\Delta x = \frac{b-a}{n} = \frac{8-2}{3} = 2$   
Endpoints  
 2 3 4 5 6 7 8  
heights  
 $f(3) = 8$   
 $f(5) = 24$   
 $f(7) = 48$

$A \approx 2(8 + 24 + 48) = 160$

$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n \Delta x \cdot f(x_i)$

$f\left(\frac{a+\Delta x_i}{2}\right)$

6. Rancher Wyatt wants to fence a new pasture using a straight river as one side of the boundary. If Rancher Wyatt has 1200 yards of fencing materials, what are the **DIMENSIONS** of the largest area of the pasture that Rancher Wyatt can enclose?

(a) 300 yards x 300 yards  
(b) 300 yards x 600 yards  
(c) 250 yards x 700 yards  
(d) 90,000 square yards  
(e) 180,000 square yards

*Handwritten work:*  
OPTIMIZATION  
 $A = x \cdot y$   
 $A = (1200 - 2y) \cdot y$   
 $A = 1200y - 2y^2$   
 $A' = 1200 - 4y$   
 $0 = 1200 - 4y$   
 $y = 300$

CONSTRAINT  
 $1200 = 2y + x$   
 $x = 1200 - 2y$   
 $x = 1200 - 2(300) = 600$





7. A particle has an acceleration given by  $a(t) = 12t$  on the interval  $[0, 10]$ . If this particle has an initial velocity of 12 meters per second and has a position of 15 meters at  $t = 1$ , find the position at  $t = 5$ .

- (a) 339 meters  
(b) 315 meters  
(c) 325 meters  
 (d) 311 meters  
(e) 301 meters

$$a(t) = 12t$$

$$v(t) = 6t^2 + C \rightarrow v(0) = C = 12$$

$$v(t) = 6t^2 + 12$$

$$s(t) = 2t^3 + 12t + D \rightarrow s(1) = 2 + 12 + D = 15$$

$$s(t) = 2t^3 + 12t + 1 \quad D=1$$

$$s(5) = 250 + 60 + 1$$

8. Let  $f$  be a differentiable function such that  $f(3) = 1$  and  $f'(3) = -3$ . If  $h(x) = \frac{2f(x)}{x^2 + 1}$ , find  $h'(3)$ .

- (a)  $-\frac{72}{100}$   
(b)  $-\frac{48}{100}$   
(c)  $\frac{72}{100}$   
(d)  $-\frac{72}{10}$   
(e)  $\frac{48}{10}$

$$h'(x) = \frac{(x^2+1) \cdot 2f'(x) - 2f(x) \cdot (2x)}{(x^2+1)^2}$$

$$h'(3) = \frac{10 \cdot 2 \cdot (-3) - 2(1) \cdot 6}{10^2} = \frac{-72}{100}$$

9. Find the 4003<sup>rd</sup> derivative of  $g(x) = 2 \sin(5x)$ .

- ~~(a)~~  $2 \cdot 5^{4003} \cos(5x)$   $\rightarrow g'(x) = 2 \cdot 5 \cdot \cos(5x)$   
 (b)  $-2 \cdot 5^{4003} \sin(5x)$   $\rightarrow g'(x) = -2 \cdot 5^2 \sin(5x)$   
~~(c)~~  $2 \cdot 5^{4003} \sin(5x)$   $\rightarrow g''(x) = -2 \cdot 5^3 \cos(5x)$   
~~(d)~~  $2^{4003} \cdot 5^{4003} \sin(5x)$   $g''(x) = 2 \cdot 5^4 \sin(5x)$   
 (e)  $-2 \cdot 5^{4003} \cos(5x)$

$$4 \sqrt[4]{400^3} \quad \text{R3}$$



10. Sand is being dropped at a rate of  $10 \text{ ft}^3/\text{min}$  onto a cone-shaped pile. If the height of the pile is always twice the base radius, at what rate is the height increasing when the pile is 8 ft high? Recall the volume formula for a cone is  $V = \frac{\pi}{3}r^2h$ .

(a)  $\frac{5}{64\pi} \text{ ft/min}$       $V = \frac{1}{3}\pi(\frac{1}{2}h)^2 \cdot h$   
 $= \frac{1}{3}\pi \frac{1}{4}h^2 \cdot h$   
 $= \frac{1}{12}\pi h^3$

(b)  $\frac{5}{8\pi} \text{ ft/min}$       $V = \frac{1}{3}\pi r^2 h$   
 $V = \frac{1}{3}\pi r^2 \cdot (2r)$   
 $V = \frac{2}{3}\pi r^3$

(c)  $\frac{5}{32\pi} \text{ ft/min}$       $\frac{dV}{dt} = \left(\frac{1}{4}\right)\pi h^2 \frac{dh}{dt}$

(d)  $\frac{10}{9\pi} \text{ ft/min}$       $\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$

(e)  $\frac{10}{27\pi} \text{ ft/min}$       $10 = 2\pi(4)^2 \cdot \frac{dr}{dt}$   
 $\frac{10}{16\pi} = \frac{dr}{dt}$       $\frac{10}{32\pi} = \frac{dr}{dt} \Rightarrow \frac{dh}{dt} = \frac{20}{32\pi}$

$h = 2r \rightarrow \frac{dh}{dt} = 2 \frac{dr}{dt}$   
 $\frac{1}{2}h = r$   
 $\frac{1}{2} \frac{dh}{dt} = \frac{dr}{dt}$

11. Find the value  $c$  that satisfies the conclusion of the Mean Value Theorem for the function  $f(x) = -3x^2 + 5x + 5$  on the interval  $[0, 3]$ .

(a)  $\frac{17}{6}$   
 (b) 3  
 (c)  $\frac{3}{2}$   
 (d) 0  
 (e)  $\frac{17}{18}$

If  $\left[ \begin{array}{l} \text{Continuous on } [0,3] \\ \text{differentiable on } (0,3) \end{array} \right.$   
 Then  $\rightarrow \frac{f(b)-f(a)}{b-a} = f'(c)$

$\frac{f(3)-f(0)}{3-0} = \frac{-7-5}{3} = -4$   
 $f' = -6x + 5$   
 $-6x + 5 = -4$   
 $-6x = -9$   
 $x = \frac{3}{2}$

12. Use logarithmic differentiation to find the derivative of  $f(x) = \frac{(x^3 + 2x)^{400}}{(1+x)^{300}}$ .

(a)  $f'(x) = \left[ \frac{400(3x^2 + 2)}{x^3 + 2x} - \frac{300}{1+x} \right] \cdot \frac{(x^3 + 2x)^{400}}{(1+x)^{300}}$

(b)  $f'(x) = \left[ \frac{400}{x^3 + 2x} - \frac{300}{1+x} \right] \cdot \frac{(x^3 + 2x)^{400}}{(1+x)^{300}}$

(c)  $f'(x) = \frac{400(3x^2 + 2)}{x^3 + 2x} - \frac{300}{1+x}$

(d)  $f'(x) = \frac{400(3x^2 + 2)(x^3 + 2x)^{398}}{(1+x)^{300}} - \frac{300(x^3 + 2x)^{400}}{(1+x)^{301}}$

(e)  $f'(x) = \frac{100(x^3 + 2x)^{398}(9x^3 + 12x^2 + 2x + 8)}{(1+x)^{301}}$

$\ln(f(x)) = \ln\left(\frac{(x^3 + 2x)^{400}}{(1+x)^{300}}\right)$   
 $= 400 \ln(x^3 + 2x) - 300 \ln(1+x)$   
 $\frac{1}{f(x)} \cdot f'(x) = 400 \cdot \frac{3x^2 + 2}{x^3 + 2x} - 300 \cdot \frac{1}{1+x}$   
 $f'(x) = \left( \quad \right) \cdot f(x)$



13. Find the  $t$ -values where the tangent line to the following parametrically defined curve is horizontal or vertical.

$$x = 2t^3 - t^2 + 6 \quad \text{and} \quad y = -t^3 + \frac{9}{2}t^2 - 6t \quad \frac{y'}{x'} = \frac{-3t^2 + 9t - 6}{6t^2 - 2t}$$

$$x' = 6t^2 - 2t \quad y' = -3t^2 + 9t - 6$$

- (a) horizontal tangents occur at  $t = 1, 2$ ; vertical tangents occur at  $t = 0, \frac{1}{3}$   
 (b) horizontal tangents occur at  $t = 0, \frac{1}{3}$ ; vertical tangents occur at  $t = 1, 2$   
 (c) horizontal tangents occur at  $t = \frac{2}{3}, 1$ ; vertical tangents occur at  $t = 0$   
 (d) horizontal tangents occur at  $t = 0$ ; vertical tangents occur at  $t = \frac{2}{3}, 1$   
 (e) horizontal tangents occur at  $t = 1$ ; there are no vertical tangents

Horizontal:  
 $-3t^2 + 9t - 6 = 0$   
 $t^2 - 3t + 2 = 0$   
 $(t-2)(t-1) = 0$   
 $t = 2, t = 1$

Vertical:  
 $2t(3t-1) = 0$   
 $t = 0, \frac{1}{3}$

14. Which of the following is a vector of unit length tangent to  $\langle \sqrt{10t+5}, e^{4t-8} \rangle$  at  $t = 2$ ?

- (a)  $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$   
 (b)  $\langle \frac{3}{5}, \frac{4}{5} \rangle$   
 (c)  $\langle \frac{1}{\sqrt{5}}, \frac{4}{\sqrt{5}} \rangle$   
 (d)  $\langle 1, 1 \rangle$   
 (e)  $\langle 1, \frac{4}{3} \rangle$

③ divide by magnitude  
 ① take a derivative  
 ② plug in 2

$$\vec{r}'(t) = \langle \frac{1}{2}(10t+5)^{-\frac{1}{2}} \cdot 10, 4e^{4t-8} \rangle$$

$$\vec{r}'(2) = \langle \frac{1}{2}(25)^{-\frac{1}{2}} \cdot 10, 4e^0 \rangle = \langle 1, 4 \rangle$$

$$\frac{\vec{r}'(2)}{\|\vec{r}'(2)\|} = \frac{\langle 1, 4 \rangle}{\sqrt{1^2 + 4^2}} = \frac{\langle 1, 4 \rangle}{\sqrt{17}}$$

15. Find the values of the constants  $a$  and  $b$  that make the following piecewise function differentiable everywhere:

$$f(x) = \begin{cases} ax^3 + 16x & \text{if } x < 1 \\ 5x^2 + b & \text{if } x \geq 1 \end{cases}$$

$f(x)$  continuous

$$\lim_{x \rightarrow 1^+} f(x) = 5 + b$$

$$\lim_{x \rightarrow 1^-} f(x) = a + 16$$

$$5 + b = a + 16$$

$$5 + b = 14$$

$$b = 9$$

$$f'(x) = \begin{cases} 3ax^2 + 16 & \text{if } x < 1 \\ 10x & \text{if } x \geq 1 \end{cases}$$

$$\lim_{x \rightarrow 1^+} f'(x) = 10$$

$$\lim_{x \rightarrow 1^-} f'(x) = 3a + 16$$

$$3a + 16 = 10$$

$$3a = -6$$

$$a = -2$$

- (a)  $a = 16, b = 5$   
 (b)  $a = -2, b = 9$   
 (c)  $a = -\frac{5}{3}, b = \frac{28}{3}$   
 (d)  $a = 1, b = 0$   
 (e) there is not enough information to determine  $a$  and  $b$

16. Suppose  $\int_5^9 g(x) dx = 4$ . Evaluate  $\int_5^9 (3 - 4g(x)) dx$

(a) 39  
 (b) -13  
 (c) 19  
 (d) -4  
 (e) -36

$$\begin{aligned}
 &= \int_5^9 3 dx - 4 \int_5^9 g(x) dx \\
 &= \int_5^9 3 dx - 4 \cdot 4 \\
 &= 3x \Big|_5^9 - 16 \\
 &= 12 - 16
 \end{aligned}$$

17. Let  $f(x) = \int_{\tan x}^x \frac{1}{\sqrt{4+t^3}} dt$ . Find  $f'(x)$

(a)  $f'(x) = -\frac{\tan(x)}{\sqrt{4+\tan^3(x)}} + \frac{1}{\sqrt{4+x^3}}$   
 (b)  $f'(x) = \frac{\sec^2(x)}{\sqrt{4+\tan^3(x)}} + \frac{1}{\sqrt{4+x^3}}$   
 (c)  $f'(x) = -\frac{\sec^2(x)}{\sqrt{4+\tan^3(x)}} + \frac{1}{\sqrt{4+x^3}}$   
 (d)  $f'(x) = \frac{\tan(x)}{\sqrt{4+\tan^3(x)}} + \frac{1}{\sqrt{4+x^3}}$   
 (e)  $f'(x)$  does not exist

$$f'(x) = \frac{1}{\sqrt{4+x^3}} - \frac{1}{\sqrt{4+\tan^3 x}} \cdot \sec^2 x$$

18. Evaluate  $\int_1^2 \left( \frac{9}{x^5} - \frac{2}{x} \right) dx$

(a)  $\frac{135}{64} - 2 \ln(2)$   
 (b)  $-\frac{135}{64} - \ln(2)$   
 (c)  $\frac{135}{64} + 2 \ln(2)$   
 (d)  $-\frac{135}{64} - 2 \ln(2)$   
 (e)  $\frac{135}{64} + \ln(2)$

$$\begin{aligned}
 &\left( \frac{-9}{4x^4} - 2 \ln|x| \right) \Big|_1^2 \\
 &= \frac{-9}{64} - 2 \ln(2) - \left( -\frac{9}{4} - 2 \ln(1) \right) \\
 &= \frac{-9}{64} + \frac{144}{64} - 2 \ln(2) \\
 &= \frac{135}{64} - 2 \ln(2)
 \end{aligned}$$

$9x^{-5} \rightarrow \frac{9x^{-4}}{-4}$   
 $\rightarrow \frac{9}{-4}$

$\frac{16}{144}$



19. Evaluate  $\int \left( 3x^2 - 10 + \frac{3}{x^2 + 1} \right) dx$ .

~~(a)~~  $\frac{x^3}{3} - 10x + 3 \arctan(x) + C$

~~(b)~~  $\frac{x^3}{3} - 10x + 3 \arcsin(x) + C$

(c)  $x^3 - 10x + 3 \arctan(x) + C$

(d)  $x^3 - 10x + 3 \arcsin(x) + C$

~~(e)~~  $\frac{x^3}{3} - 10x + 3 \tan(x) + C$

$\int_0^{7/3} | \quad | + \int_{7/3}^4 | \quad |$   
↑ neg/pos    neg/pos

NOT distance

20. The velocity function, in meters per second, is  $v(t) = 3t - 7$ . What is the displacement of the particle in the first four seconds it moves?

(a) 4 m

(b) -32 m

(c) 32 m

(d) 12 m

(e) -4 m

displacement =  $\int_0^4 v(t) dt$   
 $= \int_0^4 (3t - 7) dt = \left( \frac{3t^2}{2} - 7t \right) \Big|_0^4$   
 $= 24 - 28 - 0$

21. A plane is flying at 850 mph at N45°E. The wind is blowing at 30mph S60°E. Find the true direction of the plane.



(a)  $\theta = \arctan \left( \frac{425\sqrt{2} + 15\sqrt{3}}{425\sqrt{2} - 15} \right)$

(b)  $\theta = \arctan \left( \frac{425\sqrt{2} + 15\sqrt{3}}{425\sqrt{2} + 15} \right)$

(c)  $\theta = \arctan \left( \frac{15 - 425\sqrt{2}}{15\sqrt{3} + 425\sqrt{2}} \right)$

(d)  $\theta = \arctan \left( \frac{425\sqrt{2} - 15}{425\sqrt{2} + 15\sqrt{3}} \right)$

(e)  $\theta = \arctan \left( \frac{15\sqrt{3} + 425\sqrt{2}}{15 - 425\sqrt{2}} \right)$

$\vec{p} = \langle 850 \cos(45^\circ), 850 \sin(45^\circ) \rangle$   
 $= \langle 425\sqrt{2}, 425\sqrt{2} \rangle$

$\vec{w} = \langle 30 \cos(30^\circ), -30 \sin(30^\circ) \rangle$   
 $= \langle 15\sqrt{3}, -15 \rangle$

$\vec{t} = \langle 425\sqrt{2} + 15\sqrt{3}, 425\sqrt{2} - 15 \rangle$

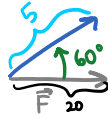
$\theta = \arctan \left( \frac{y}{x} \right) = \arctan \left( \frac{425\sqrt{2} - 15}{425\sqrt{2} + 15\sqrt{3}} \right)$





22. A horizontal force of 20 pounds is acting on a box as it is pushed up a ramp that is 5 feet long and inclined at an angle of  $60^\circ$  above the horizontal. Find the work done on the box.

- (a) 50 ft-lb  
(b)  $50\sqrt{3}$  ft-lb  
(c)  $50\sqrt{2}$  ft-lb  
(d) 100 ft-lb  
(e) 10 ft-lb

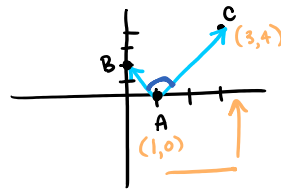


$$\begin{aligned} \vec{F} \cdot \vec{D} \\ \text{Work} &= \|\vec{F}\| \cdot \|\vec{D}\| \cdot \cos \theta \\ &= 20 \cdot 5 \cos(60^\circ) \\ &= 50 \text{ ft-lb} \end{aligned}$$

$$\vec{F} \cdot \vec{D} = \|\vec{F}\| \cdot \|\vec{D}\| \cdot \cos \theta$$

23. Given the points  $A(1, 0)$ ,  $B(0, 2)$  and  $C(3, 4)$ , find the angle,  $\theta$ , located at the vertex  $A$ . That is,  $\angle BAC$ .

- (a)  $\theta = \arccos\left(\frac{3}{5}\right)$   
(b)  $\theta = \arccos\left(-\frac{1}{\sqrt{65}}\right)$   
(c)  $\theta = 180^\circ$   
(d)  $\theta = \arccos\left(\frac{1}{\sqrt{65}}\right)$   
(e)  $\theta = \arccos\left(\frac{3}{\sqrt{17}}\right)$



$$\begin{aligned} \vec{AB} &= \langle -1, 2 \rangle \\ \vec{AC} &= \langle 2, 4 \rangle \end{aligned}$$

$$\begin{aligned} \cos \theta &= \frac{-2 + 8}{\sqrt{5} \sqrt{20}} = \frac{6}{10} \\ \theta &= \arccos\left(\frac{3}{5}\right) \end{aligned}$$

24. Find the slope of the tangent line to the graph  $x^2y^2 - 3y = 0$  at the point  $(1, -3)$ .

- (a)  $-\frac{5}{2}$   
(b)  $\frac{5}{2}$   
(c)  $-8$   
(d) 2  
(e)  $-2$

$$x^2 \cdot 2y \cdot y' + y^2 \cdot 2x - 3y' = 0$$

$$y'(2x^2y - 3) = -2xy^2$$

$$y' = \frac{-2xy^2}{2x^2y - 3}$$

$$y' = \frac{-18}{-6-3} = \frac{-18}{-9} = 2$$



25. Which of the following graphs would match the parametric equations?

$x = 3t^2$  and  $y = 2 - 5t$

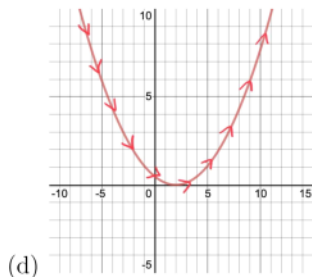
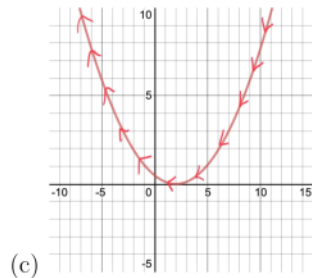
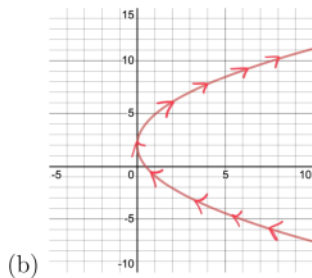
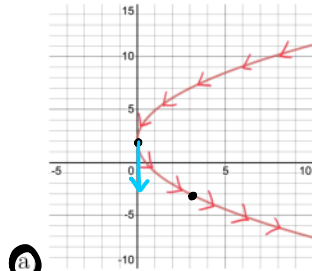
$x' = 6t$

t	x	y
0	0	2
1	3	-3
2	12	-8

$\frac{y-2}{-5} = \frac{-5t}{-5}$   
 $t = \frac{y-2}{-5}$

$x = 3\left(\frac{y-2}{-5}\right)^2$

$x' = 6t$        $y' = -5$   
↑  
 $\langle 0, -5 \rangle$

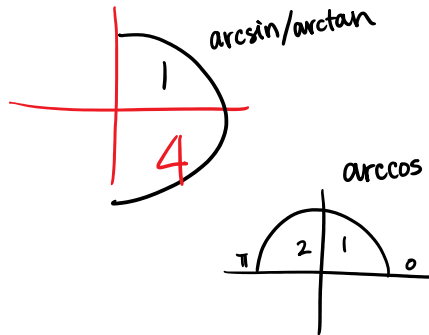


(e) None of the listed answers.



26. What is  $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ ?

- (a)  $\frac{5\pi}{6}$
- (b)  $-\frac{\pi}{6}$
- (c)  $-\frac{\pi}{3}$
- (d)  $\frac{2\pi}{3}$
- (e)  $-\frac{5\pi}{6}$



27. Calculate  $\lim_{x \rightarrow 3^+} \frac{x^2 - 2x - 7}{x^2 - 5x + 6} = \lim_{x \rightarrow 3^+} \frac{-4}{(x-3)(x-2)} = \frac{-4}{\text{tiny pos.}} = -\infty$

- (a)  $-\infty$
- (b)  $\infty$
- ~~(c) 1~~
- ~~(d) -4~~
- ~~(e)  $-\frac{7}{6}$~~

28. Use linear approximation to estimate  $\sqrt[3]{27.2}$

- (a)  $\frac{801}{270}$
- (b)  $\frac{1}{135}$
- (c)  $\frac{406}{135}$
- (d)  $\frac{402}{135}$
- (e)  $\frac{803}{270}$

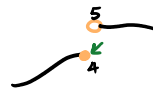
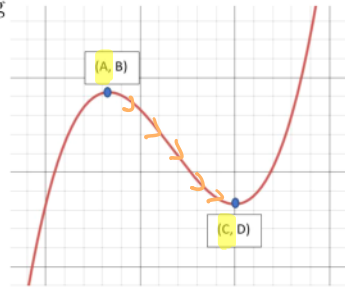
$a = 27 \quad f(x) = \sqrt[3]{x}$   
 $L(x) = f(a) + f'(a)(x-a)$   
 $y - y_1 = m(x - x_1)$   
 $y - 3 = \frac{1}{27}(x - 27) \leftarrow y = \frac{1}{27}(27.2 - 27) + 3$   
 $y = \frac{1}{27}x - 1 + 3 = \frac{1}{27}\left(\frac{1}{5}\right) + 3$   
 $y = \frac{1}{27}x + 2 = \frac{1}{135} + 3$   
 $= \frac{1 + 405}{135}$

$f(27) = 3$   
 $f'(x) = \frac{1}{3}x^{-2/3}$   
 $= \frac{1}{3(\sqrt[3]{27})^2}$   
 $= \frac{1}{27}$



29. Given the graph of  $f(x)$ , on which of the following interval(s) is  $g'(x)$  negative if  $g(x) = 3f(x)$ ?

- (a)  $(-\infty, A), (C, \infty)$
- (b)  $(-\infty, B), (D, \infty)$
- (c)  $(A, C)$**
- (d)  $(B, D)$  only
- (e) none of these



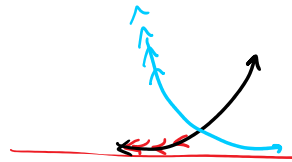
30. Given  $f(x) = \begin{cases} x+2 & \text{if } x < 2 \\ x^2 & \text{if } x = 2 \\ 5 & \text{if } x > 2 \end{cases}$ . Which of the following statements is true?

- (a)  $f(x)$  is continuous from the left at  $x = 2$ .**
- (b)  $f(x)$  is continuous from the right at  $x = 2$ .
- ~~(c)  $f(x)$  is continuous at  $x = 2$ .~~
- (d) None of these is true.
- ~~(e)  $f(x)$  is not continuous at  $x = 2$  because  $\lim_{x \rightarrow 2} f(x)$  exists but does not equal  $f(2)$ .~~

No jump

31. Compute  $\lim_{x \rightarrow -\infty} \frac{5e^{2x} - 8e^{-3x}}{3e^{2x} + 2e^{-3x}} = -\frac{8}{2}$

- (a) 0
- (b)  $\frac{5}{3}$
- (c) -4**
- (d)  $-\infty$
- (e)  $\infty$





32. Calculate  $\lim_{x \rightarrow \infty} [\ln(1 + 2x) - \ln(2 + x)]$ .

(a) 0

(b) 1

(c)  $\ln(2)$

(d)  $\infty$

(e)  $-\infty$

$$= \lim_{x \rightarrow \infty} \ln\left(\frac{1+2x}{2+x}\right) = \ln(2)$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \ln\left(\frac{2}{1}\right)$$

33. The domain of  $f(x)$  is all real numbers and  $f''(x) = 3x(x^2 - 16)(x - 4)$ . Give the  $x$ -coordinate of the inflection point(s).

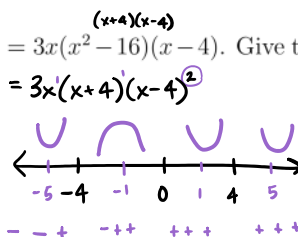
(a)  $x = 0, x = 4,$  and  $x = -4$

(b)  $x = 0$  and  $x = -4$  only

(c)  $x = 4$  and  $x = -4$  only

(d)  $x = 0$  and  $x = 4$  only

(e)  $f(x)$  has no inflection points.



34. An object is moving according to the equation of motion  $s(t) = \cos t + \frac{1}{4}t^2$ . Find the time(s) when the acceleration is zero for  $0 \leq t \leq 2\pi$ .

(a)  $t = \frac{\pi}{3}, \frac{2\pi}{3}$

(b)  $t = \frac{\pi}{6}, \frac{5\pi}{6}$

(c)  $t = \frac{4\pi}{3}, \frac{5\pi}{3}$

(d)  $t = \frac{7\pi}{6}, \frac{11\pi}{6}$

(e)  $t = \frac{\pi}{3}, \frac{5\pi}{3}$

$$v(t) = -\sin(t) + \frac{1}{2}t$$

$$a(t) = -\cos(t) + \frac{1}{2}$$

$$0 = -\cos(t) + \frac{1}{2}$$

$$\cos(t) = \frac{1}{2}$$

35. Find the derivative of the function  $f(x) = \arcsin(e^{4x})$

(a)  $f'(x) = \frac{4e^{4x}}{\sqrt{1 - e^{8x}}}$

(b)  $f'(x) = -\frac{4e^{4x}}{\sqrt{1 - e^{8x}}}$

(c)  $f'(x) = \frac{4e^{4x}}{1 + e^{8x}}$

(d)  $f'(x) = -\frac{4e^{4x}}{1 + e^{8x}}$

(e)  $f'(x) = \frac{4e^{4x}}{1 - e^{8x}}$

$$f'(x) = \frac{1}{\sqrt{1 - (e^{4x})^2}} \cdot e^{4x} \cdot 4$$