Week-in-Review 8: Chapter K2, 3.7, 3.8
(Derivatives of Parametric equations and Applications of derivatives.)

Problem 1. Find the equation(s) of the tangent line(s) to the parametric curve given by $x = 3t^2 + 1,\ y = 2t^3 + 1,$ that pass through the point $(4, 3)$.

Problem 2. Find the slope of the tangent line to the curve given by $x = \sin(3s^2),\ y = s\ln(1-s)$.

Problem 3. Find the equation of the tangent line to the curve $x = 5e^{t^2},\ y = t^2+3,$ when $t = 2$.

Problem 4. Find the point(s) on the curve where the tangent lines are horizontal or vertical.

a) $x = t^3 - 3t^2, \; y = t^3 - 3t$

b) $x = \sin(2t), \; y = \sin(t)$

Problem 5. The path of a moving particle is defined by its position function,
$r(t) = 2t^3 - 12t^2 + 18t + 3,\ t \geq 0$. (t is in seconds and r is in meters.)

a) When is the particle at rest?

b) When is the particle moving forward? When is the particle moving backwards?

c) Find the total distance travelled by the particle in the first 5 seconds.

d) What is the diplacement of the particle after 5 seconds?

Problem 6. A laboratory experiment finds that the population of a bacteria culture grows at a rate proportional to its size. The bacteria population is found to triple every 5 hours.

a) Find a function that models how much bacteria there will be after t hours.

b) How much bacteria will the sample contain after 10 hours?

c) If the culture started with a population of 1000, after how long will the culture attain a population of $30,000$?

Problem 7. Strontium 90 is a radioactive isotope with a half life of 25 years.

a) If there are 20 mgs of the isotope today, how much of it will be left after 15 years?

b) After how many years will there be 2 mgs of the substance left?

Problem 8. Iodine-131 has a half life of 8 days. How much of a 40 gram sample wil be left after 48 hours?

Problem 9. If you have 100 grams of a radioactive isotope with a half-life of 10 years, how much of the isotope will you have left after 10 years? After 20 years? After 40 years?

Problem 10. Newton's Law of Cooling states the rate of cooling of an object is proportional to the temperature difference between the object and the temperature of the object's surroundings. A pie is taken from an oven where the temperature is 375° F and placed on a table in a room where the temperature is 75° F. After 20 minutes, the temperature of the pie is 200° F.

a) Find a formula for the temperature of the pie at time t, where t is measured in minutes.

b) After how long will the pie reach a temperature of 100° F?

Problem 11. A spherical balloon is being inflated. Find the rate at which the volume of the balloon is changing with respect to the radius when the radius of the balloon is 2 inches and when the radius is 4 inches.

Problem 12. A stone dropped in a pond sends out a circular ripple whose radius increases at a constant rate of 4 ft/sec. How rapidly is the area enclosed by the ripple increasing after 12 seconds?