WIR 4

WEEK-IN-REVIEW 4
(LIMITS AT INFINITY (2.6), RATES OF CHANGE (2.7, 2.8))

$\lim\limits_{x\to\pm\infty} \left\{ \begin{array}{l}\text{term with} \\ \text{largest} \\ \text{power of } x\end{array} \right\}$

Problem 1. Find the following limits, if they exists.

(a) $\lim\limits_{x\to\infty} \dfrac{4x^2+3x+5}{2-x-5x^2}$. $= \lim\limits_{x\to\infty} \dfrac{4\cancel{x^2}}{-5\cancel{x^2}} = -\dfrac{4}{5}$

HA @ $y = -4/5$ as $x \to \infty$

(b) $\lim\limits_{x\to 5} \dfrac{2x^2-13x+15}{x^2-3x-10}$. $\xrightarrow[x=5]{\text{plug in}}$ $\dfrac{2(25)-13(5)+15}{25-3(5)-10} = \dfrac{0}{0}$ indeterminate

$\lim\limits_{x\to 5} \dfrac{\cancel{(x-5)}(2x-3)}{\cancel{(x-5)}(x+2)} = \dfrac{10-3}{5+2} = \dfrac{7}{7} = 1$

Ans.

$\to$ VA @ $x=-2$, hole @ $x=5$

$2x^2-13x+15$

$x^2-13x+30$

$(x-\frac{10}{2})(x-\frac{3}{2})$

$(x-5)\underbrace{(x-\frac{3}{2})}_{\times 2}$

$(x-5)(2x-3)$

(c) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{4x^2+3x+1}}{7x-3}$. $= \dfrac{\sqrt{4x^2}}{7x} = \dfrac{\sqrt{4}\sqrt{x^2}}{7x} = \dfrac{2|x|}{7x}$

$\sqrt{x^2} = |x| = \begin{cases} -x, & x<0 \quad (x\to-\infty) \\ x, & x\geq 0 \quad (x\to\infty)\end{cases}$

$\dfrac{2(-x)}{7x} = -\dfrac{2}{7}$ Ans.

HA @ $y = -2/7$ as $x \to (-\infty)$

$2^+ \sim 2.00001$

(d) $\lim_{x\to 2^+} \frac{2x}{4-x^2}$. $\xrightarrow[2^+]{\text{plug in}}$ $\frac{2(2^+)}{4-(2^+)^2} = \frac{4^+}{4-4^+} = \frac{4^+}{-0^+} = \boxed{-\infty}$ Ans.

$4^- = 3.9999$ $\quad$ $4^+ = 4.0001$

$x=4$

$\sqrt{x^2+3x+1} \xrightarrow{\text{sig. term}} \sqrt{x^2} = |x| \sim x'$

(e) $\lim_{x\to\infty} \left(\sqrt{x^2+3x+1} - x\right)$.

$\hookrightarrow |x| - x \sim (\infty - \infty) \to$ indeterminate

conjugate $\sqrt{x^2+3x+1} + x$.

$$\left(\frac{\sqrt{x^2+3x+1} - x}{1}\right)\left(\frac{\sqrt{x^2+3x+1} + x}{\sqrt{x^2+3x+1} + x}\right) = \frac{\left(\sqrt{x^2+3x+1}\right)^2 - (x)^2}{\left(\sqrt{} + x\right)}$$

$$\frac{3\cancel{x}}{2\cancel{x}} \Leftarrow \frac{3x}{|x|+x} \Leftarrow \lim_{x\to\infty} \frac{\cancel{x^2}+3x+1-\cancel{x^2}}{\left(\sqrt{x^2+3x+1}+x\right)}$$

$\hookrightarrow \frac{3}{2}$ Ans.

$\boxed{\text{HA @ } y = 3/2 \text{ as } x\to\infty}$

(f) $\lim_{x\to 0^-} \left(\frac{1}{x} - \frac{1}{|x|}\right)$.

$$|x| = \begin{cases} -x, & x<0 \\ x, & x\geq 0 \end{cases}$$

$$\lim_{x\to 0^-} \left(\frac{1}{x} - \frac{1}{(-x)}\right) = \lim_{x\to 0^-} \left(\frac{1}{x} + \frac{1}{x}\right) = \lim_{x\to 0^-} \left(\frac{2}{x}\right)$$

$$\frac{2}{0^-} = \frac{2}{-0^+} \to (-\infty)$$ Ans.

Note $0^- < 0$ ie $\boxed{0^- = -0.00001}$

$$\frac{1}{-0.0001} - \frac{1}{0.0001} = \frac{-2}{0.0001} \longrightarrow (-\infty)$$

$(a\pm b)^2 = a^2 \pm 2ab + b^2$

Use $a^2 - b^2 = (a+b)(a-b)$

(g) $\lim_{h\to 0} \frac{(5+h)^2 - 25}{h}$. $\sim \frac{25-25}{0} \sim \frac{0}{0}$ indeterminate

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$$\lim_{h\to 0} \frac{(5+h+5)(\cancel{5}+h-\cancel{5})}{h} = \lim_{h\to 0} \frac{(10+h)(\cancel{h})}{\cancel{h}}$$

FOIL: $(5+h)^2 = \cancel{25} + 10h + h^2 - \cancel{25}$

$h(10+h)$

$= 10 + 0$

$= 10$ Ans.

(h) $\lim_{x\to 7} \frac{\frac{1}{7}-\frac{1}{x}}{2x-14}$. $\sim \frac{\left(\frac{1}{7}-\frac{1}{7}\right)}{14-14} \sim \frac{0}{0}$

$\frac{x}{x}\left(\frac{1}{7}\right) - \frac{1}{x}\cdot\frac{7}{7}$

$= \frac{x-7}{7x}$

$$\frac{\left(\frac{x-7}{7x}\right)}{2(x-7)} = \lim_{x\to 7} \frac{(\cancel{x-7})}{2(\cancel{x-7})7x}$$

$= \frac{1}{98}$ Ans.

(i) $\lim_{x\to 3} \frac{2x^2-6x}{|x-3|}$. $\sim \frac{0}{0}$

Absolute value fn

$$|x-3| = \begin{cases} -(x-3) = -x+3 & , x<3 \\ (x-3) & , x \geq 3 \end{cases}$$

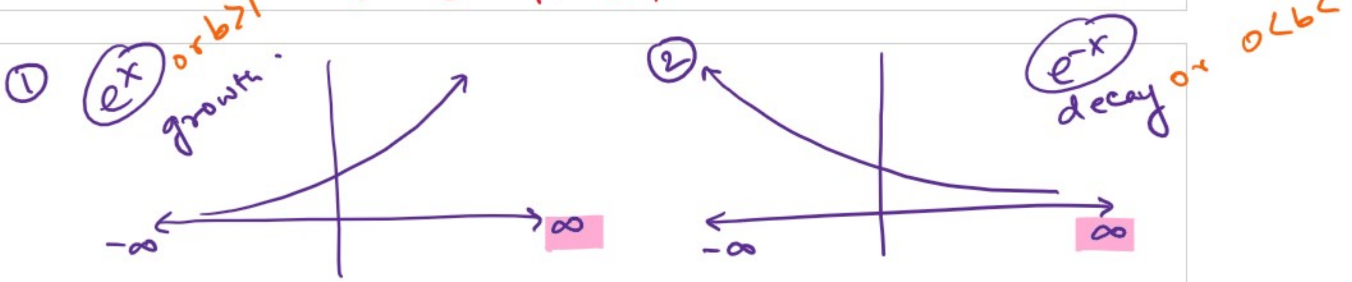
LHL

$$\lim_{x\to 3^-} \frac{2x(\cancel{x-3})}{-(\cancel{x-3})} = -6 \neq$$

RHL

$$\lim_{x\to 3^+} \frac{2x(\cancel{x-3})}{(\cancel{x-3})} = +6$$

$\therefore \lim_{x\to 3} \frac{2x^2-6x}{|x-3|}$ DNE



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(j) $\lim_{x\to\infty} \frac{1}{5+e^{-x}}$.

$\lim_{x\to\infty} e^{-x} = 0$.

$\Downarrow$

$\frac{1}{5+0} = \frac{1}{5}$ Ans.

$\lim e^x \to \infty$ $\quad \lim_{x\to\infty} e^{-x} = 0$.

(k) $\lim \frac{3e^{2x} + e^{-7x}}{}$

(k) $\lim_{x\to\infty} \frac{3e^{2x} + e^{-7x}}{4e^{2x} - 3e^{-7x}}$.

$\lim_{x\to\infty} e^{x} \to \infty \qquad \lim_{x\to\infty} e^{-x} = 0$.

$= \lim_{x\to\infty} \frac{3\cancel{e^{2x}}}{4\cancel{e^{2x}}} = \frac{3}{4}$ Ans.

(l) $\lim_{x\to-\infty} \frac{3e^{2x} + e^{-7x}}{4e^{2x} - 3e^{-7x}}$.

$\lim_{x\to(-\infty)} e^{x} = 0 \qquad \lim_{x\to(-\infty)} e^{-x} \to \infty$

$\lim_{x\to(-\infty)} \frac{\cancel{e^{-7x}}}{-3\cancel{e^{-7x}}} = \left(-\frac{1}{3}\right)$ Ans.

HA @ $y = \frac{3}{4}$ as $x \to \infty$

@ $y = \left(-\frac{1}{3}\right)$ as $x \to (-\infty)$

(m) $\lim_{x\to -3^-} e^{x/(x+3)}$.

$-3^- = -3.0001$

$x = -3$

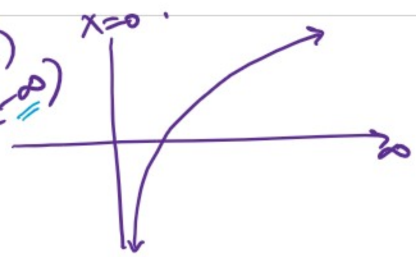
$\lim_{x\to(-3^-)} \left(\frac{x}{x+3}\right) = \frac{-3}{-3^- + 3} = \frac{-3}{-0^+} = +\infty$

$= e^{\infty} = \infty$

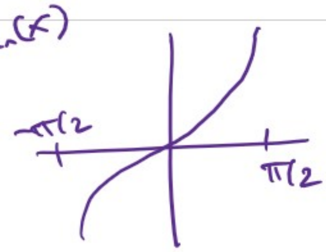
③ $\ln(x)$

$x = 0$.

$\lim_{x\to 0^+} \ln(x) = (-\infty)$



④ $\tan(x)$



$\tan\left(\frac{\pi}{2}\right) \to \infty$

$\tan\left(\frac{-\pi}{2}\right) \to (-\infty)$

(n) $\lim_{x\to\infty} [\ln(3x^6 + 1) - \ln(x^6 + 5)]$. $\sim \ln(\infty) - \ln(\infty) \sim (\infty - \infty) \leftarrow$ indeterminate

log laws: $\ln a - \ln b = \ln\left(\frac{a}{b}\right)$

$= \lim_{x\to\infty} \ln\left(\frac{3x^6 + 1}{x^6 + 5}\right) = \lim_{x\to\infty} \ln\left(\frac{3\cancel{x^6}}{\cancel{x^6}}\right) = \ln 3$ Ans.

HA @ $y = \ln(3)$ as $x \to \infty$

(o) $\lim_{x\to\infty} [\ln(3x^4 + 1) - \ln(x^6 + 5)]$. $\sim (\infty - \infty)$

$\lim_{x\to\infty} \ln\left(\frac{3x^4 + 1}{x^6 + 5}\right) = \lim_{x\to\infty} \ln\left(\frac{3x^4}{x^6}\right)$

$= \lim_{x\to\infty} \ln\left(\frac{3}{x^2}\right) = \ln(0^+)$

$(-\infty)$ Ans.

$$= \lim_{x\to\infty} \ln\left(\frac{3}{x^2}\right) = \ln(0^+)$$

$$= (-\infty) \text{ Ans.}$$

not HA.

(p) $\lim_{x\to\infty} [\ln(2^{3x} + 2)]$.

$$\lim_{x\to\infty} (2^{3x} + 2) \longrightarrow \lim_{x\to\infty} (2^{3x}) \to \infty$$

$(b=2) > 1 \to$ exp growth curve.

$$\ln(\infty) \to \infty$$

Ans.

(q) $\lim_{x\to\infty} \arctan\left(\frac{5x^2-1}{5x^2-3}\right)$.

$$\lim_{x\to\infty} \left(\frac{5x^2+1}{5x^2+3}\right) = \lim_{x\to\infty} \left(\frac{5x^2}{5x^2}\right) = 1$$

$$\arctan(1) = \pi/4 \text{ Ans.}$$

Problem 2. Given the function $f(x) - \sqrt{2x - 3}$, $\rightarrow$ $f(x+h) = \sqrt{2(x+h)+3}$

(a) Use the $\underbrace{\text{limit definition of the derivative}}$ to find $f'(x)$.

$$m = f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \qquad (a-b)\cdot(a+b) = a^2 - b^2$$

$$= \lim_{h\to 0} \frac{\sqrt{2x+2h+3} - \sqrt{2x+3}}{h} \cdot \frac{(\sqrt{2x+2h+3} + \sqrt{2x+3})}{(\sqrt{2x+2h+3} + \sqrt{2x+3})}$$

$$= \lim_{h\to 0} \frac{(\cancel{2x} + 2h + \cancel{3}) - \cancel{2x} - \cancel{3}}{h(\sqrt{2x+2h+3} + \sqrt{2x+3})}$$

$$= \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(\sqrt{2x+2h+3} + \sqrt{2x+3})}$$

$$= \frac{2}{\sqrt{2x+3} + \sqrt{2x+3}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2x+3}} = \frac{1}{\sqrt{2x+3}} = f'(x)$$

Ans.

(b) Find the $\underbrace{\text{equation of the tangent line}}$ to the function $f(x)$ at $x = 2$.

$f(x) = \sqrt{2x+3}$

$f(2) = \sqrt{4+3}$

$y = mx + b$

① Point : $(2, f(2)) = (2, \sqrt{7})$

② Slope : $m(x=2) = f'(x=2)$

$$= \frac{1}{\sqrt{2x+3}}\bigg|_{x=2} = \frac{1}{\sqrt{7}}$$

$y - y_1 = m(x - x_1)$

Ans: $\boxed{y - \sqrt{7} = \left(\frac{1}{\sqrt{7}}\right)(x-2)} = \frac{1}{\sqrt{7}}(x) - \frac{2}{\sqrt{7}}$

$$y = \left(\frac{1}{\sqrt{7}}\right)x - \left(\frac{2}{\sqrt{7}} + \sqrt{7}\right)$$

Problem 3. The position function of a moving particle is given by $f(t) - 4t^2 - 3t$, where t represents time in seconds. , position in meters

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(a) Find the average velocity of the particle from $t - 1$ to $t - 4$.

$$\text{average velocity} = \frac{f(t=4) - f(t=1)}{(4-1)}$$

$$= \frac{(4(4^2) - 3(4)) - (4-3)}{3}$$

$$= \frac{64 - 12 - 1}{3} = \frac{51}{3} = 17 \text{ m/s}$$ Ans.

$\lim_{h \to 0}$ $b = a+h$

$(b-a) = h$

$\frac{f(b) - f(a)}{b-a}$.

(b) Find the $\underbrace{\text{instantaneous velocity}}$ of the particle at time $t - 2$.

$\boxed{f(t) = 4t^2 - 3t}$

$$f'(t=2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$f(2) = 4(4) - 3(2) = 16 - 6 = 10$$

$$f(2+h) = 4(2+h)^2 - 3(2+h)$$

$$= 4(4 + 4h + h^2) - 6 - 3h$$

$$= 16 + 16h + 4h^2 - 6 - 3h$$

$$f'(2) = \lim_{h \to 0} \frac{\cancel{16} + 16h + 4h^2 - \cancel{6} - 3h - \cancel{10}}{h} = \frac{13h + 4h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(13 + 4h)}{\cancel{h}} = 13 \text{ m/s}$$ Ans.

Problem 4. Given that $f(x) = \frac{2}{5x+1}$, use the limit definition of the derivative to find $f'(x)$.

Next time!

$\boxed{x = a = (-1)}$ $f(-1) = 4$

(*) **Problem 5.** Given that the graph of a function $f(x)$ passes through the point $(-1, 4)$, and that the equation of a line tangent to $f(x)$ at this point is given by $y = 5 - 3x$, what is $\lim_{x \to (-1)} \frac{f(x) - 4}{x + 1} = ?$

$(-1, 4)$

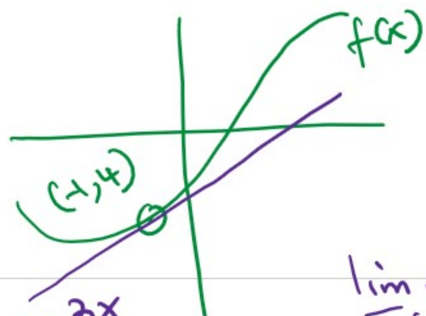
Ans:
a) 0,
b) −3,
c) 8,
d) 2,
e) 5

$$\lim_{x \to (-1)} \frac{f(x) - 4}{x + 1} = \frac{f(x) - 4}{x - (-1)} = (-3)$$

$\lim_{h \to 0} (b = a + h)$

$$= \lim_{b \to a} \frac{f(b) - f(a)}{b - a}$$

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{(a+h) - a}$$



$y = 5 - 3x$

$m = (-3)$

lim def of derivative $= \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h} = \boxed{f'(-1)}$

= "m" of tangent line

slope