Problems:

1. Find the critical points of the following functions.
   (a) $f(x) = \frac{7-x}{x+1}$
   (b) $g(x) = (x^3 - 12x)^{1/3}$
2. Find the absolute maximum and minimum of each of the following functions on the given interval.
   (a) $f(x) = \frac{1}{x}$ on [1,5].
   (b) $g(x) = -5x^3$ on [-2,4].
   (c) $h(x) = x^2 e^{-x}$ on [0,4].
3. Find the global max and global min values (if exist) of the function
$$f(x) = \begin{cases} x+5, & -4 < x \le -1 \\ 3 - x^2, & -1 < x < 3 \\ 5 - x, & 3 \le x < 4 \end{cases}$$
4. Classify the local extrema of $f(x)$ given $f'(x) = (x-3)^5(x+1)(x+7)^8$.
5. If $f'(x) = x(4x-1)^{2/3}$, find where the function is concave up. Are there any points of inflection?
6. If $f(x) = x^2 \ln\left(\frac{x}{4}\right)$, find where the function is concave up. Are there any points of inflection?
7. Sketch a graph of a continuous function where $x = -1$ is a critical point, but the function has no local extrema.
8. Sketch a graph of a continuous function where $x = 3$ is a local minimum and the function is not differentiable at $x = 3$
9. Does $f(x) = x\sin(x) + \cos(x)$ satisfies the Mean Value Theorem on $[0, 2\pi]$? Find all c that satisfies the conclusion of the Mean Value Theorem.
10. Let $P = P(t)$ be the size of a population. Suppose that P is continuous on [0,20] and differentiable on (0,20). Given $P(0) = 50$ and the growth rate satisfies $1 \le P' \le 5$, what are the max and min possible values of $P(20)$?